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"To Furnish an Elastic Currency": Banking, Aggregate Risk, and Welfare

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Abstract

The Federal Reserve Act calls upon the newly created Banks “... to furnish an elastic currency...” since such action was thought to be welfare improving during times of high currency demand. This paper considers the welfare implications of an “elastic currency” regime within the context of an overlapping generations model of fiat money that includes banks that face aggregate risk. Although in the economy’s stationary equilibrium the bank chooses to hold both fiat currency and illiquid capital in its portfolio, it would prefer to hold additional amounts of currency (were they available) during periods of high withdrawal demand. To remedy this problem, a central bank is introduced that offers zero−interest, intraperiod loans of currency via a discount window. When the central bank optimally chooses the size of the loan, it is shown that the resulting stationary equilibrium supports the economy’s “golden rule” allocation.

1. Introduction

In the official title of the act establishing the Federal Reserve System, the newly created Federal Reserve Banks are called upon "... to furnish an elastic currency ... ." As Friedman and Schwartz (1963) point out, this language was likely included in the title since the framers of the act were very much interested in finding ways to increase liquidity, and hence to lower interest rates, during periods of high currency demand such as harvest time. Having the central bank provide additional liquidity during such periods is consistent with the real-bills doctrine which recommends that the money supply vary with its demand.[1]

The goal of this paper is to investigate the welfare implications of an "elastic currency" regime within the context of a dynamic general equilibrium model of banking in which private banks are subject to random variations in the size of their withdrawal demand. It is shown that during periods of high withdrawal demand, a temporary increase in the supply of fiat currency on the part of the central bank is welfare improving. As the injection of currency is both finite and is removed from circulation at the end of the period, this policy is shown to cause a temporary fall in the rate of interest and to do so without causing unbounded money growth. Hence, the model is consistent with Friedman and Schwartz's (1963) observation that interest rates during harvest time were lower after the creation of the Federal Reserve System than before and shows that a common criticism of real-bills policies, namely Wicksellian indeterminacy, need not arise.[2]

The modeling framework utilized here begins with a monetary version of Diamond and Dybvig's (1983) banking model with the bank subject to aggregate risk regarding the magnitude of its withdrawal demand. In order to provide a relatively simple way to obtain valued fiat currency in equilibrium, the bank is placed in a three-period-lived overlapping generations framework. Similar modeling approaches are found Smith (1987), Loewy (1991, 1998), Freeman (1996, 1999), Zhou (2000), and Kahn and Roberds (2001).

In my model, the bank acquires fiat currency, reserves, and an "illiquid" real asset, capital, to support the deposits of young agents. Since the bank is assumed to be unable to observe ex ante the measure of middle-aged agents seeking to withdraw and currency dominates capital in rate of return over one period, the bank finds it optimal to finance the withdrawals of early-arriving middle-aged agents solely with currency. As it is also the case that capital dominates currency in rate of return over two periods, it is also optimal for the bank to exhaust its reserves in financing these particular withdrawals. Thus, any late-arriving middle-aged agents and all members of the same generation who wish to withdraw when old receive only capital. While having the latter receive only capital is desirable, having the former do so is not since these agents, who receive less consumption than do those who arrive earlier, are made to bear all of the risk associated with the variation in withdrawal demand.[3] Those middle-aged agents who do receive currency then exchange it for goods in the spot market for currency. This serves to transfer the currency back to the bank where it now serves as an asset supporting the deposits of the next generation.

The short-run rate of return dominance of fiat currency implies that if middle-aged withdrawal demand proves to be high, then the bank would prefer to finance (in part or in total) the withdrawals of late-arriving agents using currency were it available. One mechanism for its provision is a central bank that operates a type of discount window. Here, this policy takes the form of a zero-interest, intraperiod loan of fiat currency in exchange for (claims on) immature capital.[4] Given that the bank takes rates of return on money as given, it follows that the bank will accept however much currency the central bank is willing to offer. From the vantage point of the central bank however, a finite, optimal upper bound for the loan does

[1] See Sargent and Wallace (1982), Freeman (1996), and Sproul (2000a, b).

[2] Such criticisms appear in Mints (1945), Friedman and Schwartz (1963), Laidler (1984), and Selgin (1989), among others.

[3] That the optimal response to high withdrawal demand is a partial suspension of withdrawals is also seen in Wallace (1988, 1990), Loewy (1998), and Green and Lin (2000).

[4] Freeman (1996) shows that the optimal interest rate on such loans is zero when default risk is absent as it is here.

exist since although more currency increases the consumption of late-arriving middle-aged agents, *ceteris paribus*, it also reduces the rate of return on money which in turn reduces the consumption of all currency-holding agents. In this case, I show that the resulting stationary monetary equilibrium not only Pareto dominates the one that the bank can achieve in the absence of central bank credit, it also supports the "golden rule" allocation, namely the allocation that maximizes steady-state ex ante utility subject to the economy's feasibility constraints. The optimal central bank loan achieves this allocation by being of exactly the right size so as to eliminate the partial suspension of withdrawals that otherwise arises when withdrawal demand is high and so spreads the risk associated with variable withdrawal demand across all middle-aged agents.

While the central bank can utilize its discount window to achieve a Pareto-improving allocation, it should be noted that this result ought to be viewed as a positive, rather than a normative, implication of the model. Such a viewpoint is needed here because the paper takes the existence and liquidity-provision role of the central bank as given without justifying why the central bank exists or engages in this activity. In addition, the paper makes no claim that the private sector is unable to achieve the same Pareto-improving allocation on its own. Inasmuch as this ability would preclude the existence of a central bank, the model includes a pair of "legal restrictions" that prevent the private sector from engaging in activities that serve to duplicate what the paper takes as the given function of the central bank.

Although Freeman (1996, 1999) and Zhou (2000) consider monetary banking models similar to that used here, their approaches differ inasmuch as each considers environments in which there exists intragenerational borrowing and lending. They then focus on how a central bank can overcome the liquidity problems that arise in the clearing of nominal debt through the use of either open market operations or a discount window. While debt per se does not exist in my model, liquidity problems nevertheless arise and for reasons similar to that of Freeman and Zhou – a mismatch between the number of agents who wish to consume and the amount of funds available to finance this consumption. In contrast, Champ et al. (1996) and Williamson (1998) provide a different general equilibrium monetary model of banking, one in which the need for currency arises from depositors demanding liquidity in order to move from one location to another rather than from uncertainty over the timing of consumption as is the case here.

The remainder of this paper is organized as follows. The next section outlines the economy and the sequence of events within each period. Section 3 considers the commercial bank's problem while the existence of a stationary monetary equilibrium is the subject of Section 4. Section 5 describes the bank's problem in the presence of central bank credit and the corresponding problem of the central bank. It then establishes the existence a stationary monetary equilibrium that is the solution to the central bank's problem and therefore is also the equilibrium that will arise when the commercial bank is provided with the optimal currency injection. The section finishes by defining a "golden rule allocation" and showing that the stationary monetary equilibrium with central bank credit supports such an allocation. Section 6 concludes.

2. The Environment

2.1 Fundamentals

Consider an economy of three-period-lived overlapping generations where at each date $t \geq 1$ a continuum of ex ante identical agents (taken to be of measure 1) is born. The generation to which each agent belongs is public information. Each agent born at time t is endowed with one unit of the economy's lone consumption good during time t and no units at any other date. Following Bhattacharya and Gale (1987), Freeman (1988), Loewy (1991, 1998), Qi (1994) and others, I assume that each generation consists of two types of agents, those who consume during their second period of life (early consumers) and those who consume during their third period of life (late consumers). An agent's type is assumed to be private information with the latter not becoming revealed to the agent until the beginning of his second period of life.

Besides facing the risk of type, agents and the bank also face a form of aggregate risk. Specifically, the measure of early consumers in generation $t \geq 1$ is assumed to be a random variable, $\tilde{\lambda}_t$. To keep matters simple, $\tilde{\lambda}_t$ is assumed to have a two-point support where prob $\{\tilde{\lambda}_t = \lambda^1\} = 1 - \pi$, prob $\{\tilde{\lambda}_t = \lambda^2\} = \pi$, $0 < \pi < 1$ and $0 < \lambda^1 < \lambda^2 < 1$.

Let e_t and l_t be the time t+1 (early) and t+2 (late) consumption of an agent born at time $t \geq 1$. Since an agent's type determines when he consumes, it also determines his utility. Thus, let each agent's utility function be given by $U(e_t, l_t, \theta) = \theta \ln(e_t) + (1 - \theta)\ln(l_t)$, where $\theta = 0$ if the agent is a late consumer and $\theta = 1$ if the agent is an early consumer.[5]

The economy's single technology, capital, may be accessed either directly or indirectly through an intermediary. Per capita holdings of capital, k_t, earn a fixed gross rate of return of x when held for one period and of X when held for two periods. It is assumed that $0 < x < 1 < X$. Thus, interrupting capital prematurely is costly. Note that the inability to verify an agent's type implies that IOUs backed by such goods will never be traded.

At $t = 1$ there also exist the agents of generation 0 who are taken to be of measure 1. To simplify the discussion of equilibria below, I assume that all members of generation 0 are early consumers and hence seek to maximize their consumption, e_0. These agents are assumed to hold in aggregate $M > 0$ units of fiat currency. In what follows, I make the simplifying assumption that the supply of fiat currency is fixed save for the presence of any loans that the central bank may extend should the commercial bank request them.

Last, let p_t be the time t goods price of currency and m_t be the time t per capita stock of currency. For nonnegative values of p_t and m_t, let $q_t = p_t m_t$ define per capita real balances. If $p_t > 0$ for all $t \geq 1$, then the one-period rate of return on real balances, $R_t = p_{t+1}/p_t$ is well defined. The presence of aggregate risk implies that in equilibrium R_t is conditional on the known sequence $\{\tilde{\lambda}_s\}_{s=1}^{t-1}$ as well as on the stochastic process $\{\tilde{\lambda}_s\}_{s=t}^{\infty}$. However, because in what follows it is the realization of $\tilde{\lambda}_t$ that is most relevant to the analysis, let R_t^i, $i = 1, 2$, represent the time t rate of return on real balances when $\tilde{\lambda}_t = \lambda^i$, $i = 1, 2$.

2.2 Activity within each period

At $t = 1$, the young agents of generation 1 join the middle-aged agents of generation 0 at a location where at this and at all other dates there also exist the economy's commercial bank, its central bank, and its spot market for currency. The commercial bank then offers its optimal contract to the members of generation 1 who deposit their entire endowment into the bank.[6] Assuming, as will be the case in equilibrium, that the bank's contract implies that reserves be part of its portfolio, it enters the time 1 currency spot market on behalf of the members of generation 1 by supplying time 1 good in exchange for currency that is inelastically supplied by members of generation 0. Following trade, generation 0 agents consume. Once this concludes, all members of generation 1 remain at the location until the start of period 2 while all members of generation 0 exit the economy.

Consider next the sequence of events that occur during each date $t \geq 2$. The period begins with the arrival of generation t at the location and these agents depositing their endowment with the bank in exchange for the bank's optimal contract. Coinciding with this activity, the members of generation t-1 learn their type while the members of generation $t-2$ (which exist for $t \geq 3$) remain inactive.

Second, middle-aged agents who have learned that they are early consumers randomly and uniformly contact the bank in order to withdraw while those who have learned that they are late consumers remain inactive until the following period when they will withdraw. Third, the early consumers of generation t-1 and the late consumers of generation $t-2$ withdraw. Given that early consumers randomly and uniformly arrive at the bank and that in equilibrium late consumers will self-select to withdraw in the next period, it

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[5] Diamond and Dybvig (1983), Wallace (1988), Jacklin (1993), Temzelides (1997), and Adão and Temzelides (1998) assume instead that the rate of return is sufficiently high to induce "late consumers" to do just that rather than consuming "early." In contrast, Jacklin (1987), Jacklin and Bhattacharya (1988), Chari and Jagannathan (1988), and Alonso (1996) assume that the preferences of both types are such that they consume in both the second and third period of life.

[6] This follows from the risk-sharing abilities of the bank coupled with the inability of agents to issue capital-backed claims.

follows that the realization of $\tilde{\lambda}_{t-1}$ becomes public information as soon as the bank observes the arrival (or not) of the marginal early consumer beyond measure λ^1.[7]

Fourth, those generation *t*-2 and *t*-1 agents who withdraw during time *t* receive a payment of currency and/or capital that is consistent with the bank's optimal contract. Assuming, as will be the case in equilibrium, that some of these agents will receive some or all of their payment in the form of currency, then if $\tilde{\lambda}_{t-1} = \lambda^1$ these agents proceed to the spot market where they exchange their currency for time *t* good which is supplied by the bank using the deposits of generation *t*.[8] After the spot market clears, those agents whose withdrawal consisted in part or in whole of currency now consume as do any other members of their generation who may have received only capital.

On the other hand, if $\tilde{\lambda}_{t-1} = \lambda^2$, then once the bank determines that withdrawal demand is high, it has an incentive to borrow currency to help finance the withdrawals of late-arriving agents. Although early-arriving middle-aged agents represent a possible source of such currency and the bank may be able to identify these agents since they have already visited the bank, it is assumed that there exists a legal restriction prohibiting such loans. This implies that the bank must turn to the central bank for such a loan (should it be available). By the same token, early and late-arriving middle-aged agents may be able to identify each other (since they are of the same type and in the same location) and wish to engage in side trades prior to the opening of the spot market. Here, too, it is assumed that there exists a legal restriction prohibiting such trades. Hence, those middle-aged agents holding currency may only exchange it for goods in the spot market and once they do so, they, along with any other members of generations *t*-1 and *t*-2 holding goods, consume.

Last, after consumption is complete, activity during time *t* concludes with generation *t*-2 exiting the economy while generations *t*-1 and *t* remain at the location. If the bank had received a loan, it is repaid at this time. Table 1 illustrates the sequence in which the different events comprising time *t* occur.

3. The Bank's Problem

A key feature of the environment of this paper is the presence of fiat currency and its use as a reserve asset on the part of banks. Not only does the presence of currency add a second asset to the bank's portfolio, but it also necessitates satisfying a set of market clearing conditions as part of the equilibrium. It then follows that the analysis of both the bank's problem and the equilibrium possess a degree of complexity not seen when fiat currency is absent. Therefore, in order to try to keep the analysis as tractable as possible, I make the common, simplifying assumption that the bank arranges its portfolio on a generation-by-generation basis. Under this mutual-like structure, the time *t*+1 withdrawals made by the members of generation *t* (or *t*-1) are completely financed using only the assets that the bank holds on behalf of that specific generation. By design, this assumption simplifies matters by precluding the bank from utilizing any direct form of intergenerational transfer (although such transfers do arise via the spot market for currency).[9]

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[7] As Diamond and Dybvig (1983, Prop. 1) show, it is this inability to observe $\tilde{\lambda}_{t-1}$ ex ante that precludes the bank from providing agents with the optimal risk-sharing allocation. Green and Lin (2000) argue that Diamond and Dybvig's result may be an artifact of the limited set of contracts that the bank is permitted to offer. Assuming a finite number of agents and hence the presence of aggregate risk, Green and Lin show that when the bank optimally adjusts its payments on an agent-by-agent basis conditional upon the payments it has made previously and the payments it expects to make to any remaining agents implies that the equilibrium yields the ex ante optimal risk-sharing allocation. Hence, bank runs are precluded. Peck and Shell (2003) show that this need not be the case, however, should the preferences of early and late consumers differ and depositors not know their place in line.

[8] Since the bank holds this currency against generation *t*'s deposits and also uses it to finance their withdrawals in subsequent periods, currency serves as a reserve asset. By the same token, since capital is also part of the bank's portfolio, it can usefully be thought of as constituting illiquid loans.

[9] In contrast, Qi (1994) assumes that time *t*+1 deposits are available to finance time *t*+1 withdrawals. By the same token, this effectively leads him to view time *t* investments in the two technologies as another type of withdrawal.

Table 1

Sequence of Events within Period t

Stage	Young Agents (generation t)	Middle-aged Agents (generation t-1)	Old Agents (generation t-2)	Commercial Bank
1	Enter economy; deposit endowment with commercial bank	Present at location; learn type	Present at location	Offers optimal contract to young agents
2		Early consumers randomly and uniformly contact bank		
3		Early consumers withdraw; $\tilde{\lambda}_{t-1}$ publically observed	Late consumers withdraw	Payments made; may receive loan from central bank if $\tilde{\lambda}_{t-1} = \lambda^2$
4		If hold currency, enter spot market; after trade, all $\theta = 1$ agents consume	If hold currency, enter spot market; after trade, all $\theta = 0$ agents consume	Enters spot market; demands currency on behalf of young agents
5	Remain at common location	Remain at common location	Exit the economy	Any loan repaid to central bank

In order to make the distinction between reserves and capital meaningful, the only withdrawals that are admissible are those that can be financed using anywhere from 0 to 100% of the two assets that the bank has on hand at the time that withdrawals are made.[10] This implies that the bank must choose the share of each asset that it will use to finance withdrawals at each date. Furthermore, these shares must sum to one.

Let χ_t^1 and κ_t^1 be the shares of reserves and capital held on behalf of the members of generation t that the bank uses to finance the time t+1 withdrawals of the first measure λ^1 generation t early consumers. Should a marginal early consumer beyond measure λ^1 arrive at the bank, let χ_t^2 and κ_t^2 be the shares of this generation's reserves and capital offered to the next measure $\lambda^2 - \lambda^1$ early consumers who arrive at the bank. Beyond measure λ^2 early consumers, the bank is assumed to fully suspend withdrawals. This in turn implies that generation t late consumers will self-select to postpone their withdrawals. Hence, the observation (or not) of this marginal early consumer is equivalent to observing the realization of $\tilde{\lambda}_t$.

In addition to the reserves and capital that the bank holds at time t+1 on behalf of generation t, at this date it also holds the reserves and capital of generation t-1 which remain after having financed that generation's time t withdrawals. Since the quantity of the latter depended upon the realization of $\tilde{\lambda}_{t-1}$, so too does the amount of reserves and capital available at time t+1 to finance generation t-1's withdrawals. In particular, if $\tilde{\lambda}_{t-1} = \lambda^1$, then at time t+1 the bank has the share $1 - \chi_{t-1}^1$ of generation t-1's reserves and $1 - \kappa_{t-1}^1$

[10] In other words, the bank cannot offer agents withdrawals that require that it issue more than the quantity of currency or capital it has on hand by "transforming" one asset into the other.

of their capital available to finance their withdrawals. On the other hand, if $\tilde{\lambda}_{t-1} = \lambda^2$, then the available shares of these assets fall to $1 - \chi_{t-1}^1 - \chi_{t-1}^2$ and $1 - \kappa_{t-1}^1 - \kappa_{t-1}^2$.

Since the consumption of a generation t early consumer can potentially vary with the realization of $\tilde{\lambda}_t$ (as it can affect both the shares of reserves and capital paid out as well as the rate of return on real balances), let e_t^{2h} be the time t+1 consumption of a generation t early consumer when $\tilde{\lambda}_t = \lambda^2$ and the agent is among the first (h = f) measure λ^1 agents to withdraw or among the second (h = s) measure $\lambda^2 - \lambda^1$ agents to withdraw. Analogously, let $e_t^{1\mathrm{f}}$ be the time t+1 consumption of a generation t early consumer when $\tilde{\lambda}_t = \lambda^1$ (in which case h = f must necessarily hold). Similarly, the consumption of a generation t late consumer depends upon the realizations of both $\tilde{\lambda}_t$ and $\tilde{\lambda}_{t+1}$ since the former determines both the shares of reserves and capital that are available and R_t^i while the latter determines (among other things) R_{t+1}^i. To this end, let l_t^{ij} be the time t+2 consumption of a generation t late consumer when $\tilde{\lambda}_t = \lambda^i$ and $\tilde{\lambda}_{t+1} = \lambda^j$, $i, j = 1, 2$.

The bank is taken to be a zero-profit, zero-cost institution. Given its mutual-like structure, its objective is to maximize the ex ante expected utility of its depositors at each date. Hence, for each date $t \geq 1$, the objective of the bank $\boldsymbol{U}_t^{\boldsymbol{B}}$, can be written as

$$\boldsymbol{U}_t^{\boldsymbol{B}} = (1 - \pi)\left[\lambda^1 u(e_t^{1\mathrm{f}}) + (1 - \lambda^1)\left\{(1 - \pi)u(l_t^{11}) + \pi u(l_t^{12})\right\}\right] +$$

$$\pi\left[\lambda^1 u(e_t^{2\mathrm{f}}) + (\lambda^2 - \lambda^1)u(e_t^{2\mathrm{s}}) + (1 - \lambda^2)\left\{(1 - \pi)u(l_t^{21}) + \pi u(l_t^{22})\right\}\right]. \tag{1}$$

Given the timing of events portrayed in Table 1, the mutual-like structure of the bank, and the feasibility constraints discussed above, at each date $t \geq 1$ the bank solves the following problem:

Problem *B*1 *Choose* $\{e_t^{1\mathrm{f}},\ e_t^{2\mathrm{f}},\ e_t^{2\mathrm{s}},\ l_t^{ij},\ \chi_t^i,\ \kappa_t^i,\ (i, j = 1, 2),\ q_t,\ k_t\}$ *to maximize* $\boldsymbol{U}_t^{\boldsymbol{B}}$ *subject to*

$$\lambda^1 e_t^{1\mathrm{f}} = \chi_t^1 R_t^1 q_t + \kappa_t^1 x k_t, \tag{2}$$

$$\lambda^1 e_t^{2\mathrm{f}} = \chi_t^1 R_t^2 q_t + \kappa_t^1 x k_t, \tag{3}$$

$$(\lambda^2 - \lambda^1)e_t^{2\mathrm{s}} = \chi_t^2 R_t^2 q_t + \kappa_t^2 x k_t, \tag{4}$$

$$(1 - \lambda^1)l_t^{1j} = (1 - \chi_t^1) R_t^1 R_{t+1}^j q_t + (1 - \kappa_t^1) X k_t, \quad j = 1, 2, \tag{5}$$

$$(1 - \lambda^2)l_t^{2j} = (1 - \chi_t^1 - \chi_t^2)R_t^2 R_{t+1}^j q_t + (1 - \kappa_t^1 - \kappa_t^2)Xk_t, \quad j = 1, 2, \tag{6}$$

$$q_t + k_t = 1, \tag{7}$$

$$0 \leq \chi_t^i, \kappa_t^i \leq 1, \quad i = 1, 2; \quad 0 \leq \chi_t^1 + \chi_t^2 \leq 1; \quad 0 \leq \kappa_t^1 + \kappa_t^2 \leq 1, \tag{8}$$

$$\text{Generation } t\text{'s Self-Selection Constraint}, \tag{9}$$

taking rates of return on real balances as given.

Eqs. (2) and (3) illustrate how the bank finances the withdrawals of the first measure λ_1 generation t early consumers. In particular, because the bank cannot observe $\tilde{\lambda}_t$ ex ante, the same shares of reserves and capital must be offered to each regardless of its eventual realization. Eq. (4) shows how the withdrawals of the remaining measure $\lambda^2 - \lambda^1$ early consumers are financed should $\tilde{\lambda}_t = \lambda^2$. Eqs. (5) and (6) describe how the bank finances the withdrawals of generation t late consumers. In each case, the bank must use the stocks of reserves and capital that are still available after generation t's withdrawals in the previous period have been completed. The consumption of late consumers is also dependent upon the realization of $\tilde{\lambda}_{t+1}$ which determines (in part) R^j_{t+1}. Eqs. (7) and (8) describe how the bank allocates its time t deposit plus the admissibility restrictions on the shares.

Consider next generation t's self-selection constraint. Since the early consumers of generation t never delay their withdrawal, only that generation's late consumers require such a restriction. Specifically, late consumers will self-select to withdraw in the period in which they consume when their expected utility from doing so under the bank's contract exceeds their utility from withdrawing prematurely and using the proceeds to finance consumption in the next period. However, as in Diamond and Dybvig (1983), risk sharing plus the presence of a total suspension of withdrawals implies that the constraint does not bind ex ante. In light of this, a derivation of the constraint is available from the author upon request.

4. Equilibrium

Given the stationarity inherent in the economy's structure, it is natural to consider the existence of a stationary equilibrium and, in particular, of a stationary monetary equilibrium.[11] In order to define and then prove the existence and uniqueness of such an equilibrium, it is first necessary to consider formally the market for real balances. At each date t the rate of return on real balances must be such that the demand for real balances on behalf of the members of generation t must equal the supply of real balances on the part of the early consumers of generation t-1 and (for $t \geq 3$) the late consumers of generation t-2.[12] However, because the measure of early consumers can vary, market clearing varies with the realizations of $\tilde{\lambda}_{t-1}$ (for $t \geq 2$) and $\tilde{\lambda}_{t-2}$ (for $t \geq 3$).

Given that the members of generation 0 supply $M > 0$ units of currency in aggregate at $t = 1$, market clearing at that date requires that

$$q_1 = p_1 M. \tag{10}$$

Since there may be either measure λ^1 or λ^2 early consumers in generation 1, the economy's two legal restrictions imply that at $t = 2$ one of the following two market clearing conditions must be satisfied,

$$q_2 = \chi_1^1 R_1^1 q_1, \tag{11}$$

$$q_2 = (\chi_1^1 + \chi_1^2) R_1^2 q_1. \tag{12}$$

Similarly, one of the following four expressions must hold for all $t \geq 3$,

[11] Admittedly, there may also exist non-stationary monetary equilibria. In any such equilibrium, the rate of return on real balances falls over time to its lower bound of x. As it does so, the bank substitutes capital for real balances and the economy approaches a nonmonetary equilibrium in which all saving takes the form of capital. Given the additional levels of complexity such equilibria entail, they are not considered here.

[12] This assumes, as is the case, that the self-selection constraint is non-binding in equilibrium.

$$q_t = \chi^1_{t-1} R^1_{t-1} q_{t-1} + (1 - \chi^1_{t-2}) R^1_{t-2} R^1_{t-1} q_{t-2}, \tag{13}$$

$$q_t = \chi^1_{t-1} R^1_{t-1} q_{t-1} + (1 - \chi^1_{t-2} - \chi^2_{t-2}) R^2_{t-2} R^1_{t-1} q_{t-2}, \tag{14}$$

$$q_t = (\chi^1_{t-1} + \chi^2_{t-1}) R^2_{t-1} q_{t-1} + (1 - \chi^1_{t-2}) R^1_{t-2} R^2_{t-1} q_{t-2}, \tag{15}$$

$$q_t = (\chi^1_{t-1} + \chi^2_{t-1}) R^2_{t-1} q_{t-1} + (1 - \chi^1_{t-2} - \chi^2_{t-2}) R^2_{t-2} R^2_{t-1} q_{t-2}. \tag{16}$$

Eq. (11) provides the t = 2 market clearing condition when $\tilde{\lambda}_1 = \lambda^1$ while eq. (12) provides the expression when $\tilde{\lambda}_1 = \lambda^2$. Likewise, for $t \geq 3$, eqs. (13) - (16) provide the relevant market clearing conditions as $(\tilde{\lambda}_{t-2}, \tilde{\lambda}_{t-1}) = (\lambda^1, \lambda^1)$, (λ^2, λ^1), (λ^1, λ^2), or (λ^2, λ^2) respectively. In each case, the supply of real balances varies to reflect the amounts of reserves that the bank issues to generation t-1 early consumers and to generation t-2 late consumers. Naturally, these quantities reflect the size of middle-aged withdrawal demand during both the current and the previous period.

With eqs. (10) - (16) in hand, a stationary monetary equilibrium (SME) may now be defined.

Definition 1 *Given* $M > 0$ *and the stochastic process* $\{\tilde{\lambda}_t\}_{t=1}^{\infty}$, *a* SME *consists of scalars which are strictly positive,* (p_1, R^1, R^2)*; nonnegative,* $(e^{1f}, e^{2f}, e^{2s}, l^{11}, l^{12}, l^{21}, l^{22})$*; and bounded between zero and one,* $(q, k, \chi^1, \chi^2, \kappa^1, \kappa^2)$*; such that both* Problem $B1$ *is solved and the currency market clears for every* $t \geq 1$.

I can now state and prove the paper's first result, namely that given logarithmic utility and the mutual-like structure of the bank, there exists a unique stationary monetary equilibrium if and only if a mild regularity condition on the parameters is satisfied.

Proposition 1 *There exists a unique* SME *if and only if* $1 \geq \pi/x + (1 - \pi)/X$.

(The proof of this and all subsequent propositions appear in the appendix.)

To establish Proposition 1, I propose a set of equilibrium values for the model's endogenous variables and then show that if and only if the condition given in the proposition is satisfied, then these are values are indeed the unique SME. In order to formulate the proposed values, it is first necessary to observe that in order for the currency market to clear in any SME, it must be that case that $R^1 = R^2 = 1$ and $\chi^2 = 0$ (where stationarity permits the time argument to be dropped). Second, since in equilibrium the rate of return on money exceeds x, it is optimal for the bank to pay currency to as many early consumers as possible and, by the same token, to pay immature capital to as few of them as possible. The former implies that $\chi^1 + \chi^2 = 1$ while the latter, along with the random arrival of depositors, implies that $\kappa^1 = 0$. Furthermore, because market clearing requires that $\chi^2 = 0$, we must also have that $\chi^1 = 1$. With all the currency optimally paid out to early-arriving middle-aged agents, it follows that late-arriving middle-aged agents receive only immature capital so that $\kappa^2 > 0$. Since the utility function is assumed to be logarithmic, solving for the equilibrium values of q and κ^2 under these conditions is straightforward. The condition of the proposition is then invoked to guarantee that χ^2 is indeed equal to zero and therefore that χ^1 is indeed equal to one. As for the necessity of the condition, since any stationary monetary equilibrium must have $R^1 = R^2 = 1$, $\chi^1 = 1$, and $\chi^2 = 0$, log utility implies that when the first-order conditions for χ^1 and χ^2 are evaluated at the proposed values, then the condition given in the proposition must hold.

As for the condition given in the proposition itself, note that it can be rearranged to read $x(X - 1)/(X - x) \geq \pi$. Therefore, this necessary and sufficient condition for existence imposes an upper bound on the probability of middle-aged withdrawal demand being high. Consider, then, the implications of an increase in π. In this case, the expected marginal utility of late-arriving middle-aged agents rises reflecting the increased chance that such agents will seek to withdraw. Since market clearing implies that currency dominates immature capital in rate of return, the increase in π increases the pressure on the bank to finance at least part of the withdrawals of these late arrivers using currency. When π has increased to the point where the above condition binds, this pressure is sufficiently strong that it is now optimal for the bank to trade off an increase in χ^2 against an equal decrease in χ^1. Once this occurs, the SME is no longer sustainable.[13]

Finally, although Proposition 1 is predicated upon logarithmic utility, it can be generalized somewhat. Let $u(c) = (c^{1-\gamma} - 1)/(1-\gamma)$ with $\gamma \in \mathcal{N}(1, \varepsilon)$. By continuity, there also exists a unique SME for any such γ. These equilibria will again exhibit $R^1 = R^2 = 1$, $\chi^1 = 1$, and $\chi^2 = \kappa^1 = 0$ and so will exist if and only if π, x, and γ are not too large and/or X is not too small.[14]

5. Central Bank Lending and Optimality

As shown in the previous section, in a SME the bank finds it optimal to use all of its reserves to finance the withdrawals of early-arriving middle-aged agents. In equilibrium, this result must hold despite that fact that the ex ante utility of depositors is strictly increasing in both χ^1 and χ^2. Consequently, were it possible to use currency to finance the withdrawals of late-arriving middle-aged agents with currency, then the ex ante utility would increase.

One mechanism to provide for such payment is to assume that the central bank operates a type of discount window.[15] In the present context, this is taken to mean that the central bank stands ready to lend fiat currency at zero interest up to a predetermined cap of its own choosing in exchange for some of the commercial bank's immature capital (or claims thereon) which serves to provide partial collateralization of the loan.[16] The bank then uses the loan, in the amount of ΔM say, and, potentially, some of its current stock of currency, M, and net-of-collateral immature capital to finance the withdrawals of late-arriving early consumers. The latter, and the early-arriving early consumers then supply $M + \Delta M$ units of currency to the spot market. Following the clearing of the spot market, at which point all outstanding units of currency have returned to the commercial bank, the latter repay the loan in exchange for its collateral.[17] Since the additional currency is removed from circulation by the end of the period, there is no measured increase in the money supply from the beginning of the period to the end. By avoiding unbounded money growth, a feature also found in Freeman (1996, 1999) and Zhou (2000), Wicksellian indeterminancy does not occur.

Let χ_t^3 be the size of the loan (measured as a percentage of the existing money supply) that the bank receives to help finance the withdrawals of the late-arriving early consumers of generation t and let $\overline{\chi}_t^3$ be the maximum loan permitted by the central bank (again measured as a percentage of the existing money

[13] Similarly, this condition will bind if the value of x is sufficiently high or the value of X is sufficiently low.

[14] As γ increases, optimal risk sharing increases. This again puts pressure on the bank to raise e^{2s} relative to e^{2f} by trading off an increase in χ^2 against an equal-sized decrease in χ^1.

[15] Recall that loans of currency from the early-arriving middle-aged to the bank, which constitute another mechanism, are prohibited.

[16] This policy is a hybrid of what is currently used by the central banks of the EMU and the U.S. The former charge no nominal interest on such loans but require full collateralization. The latter imposes a cap on the amount of the loan and charges interest on that part which exceeds 10% of a bank's risk-based capital. In practice, such charges are quite small or zero. See Zhou (2000).

[17] The bank will always do so since it needs the collateral to finance the withdrawals of late consumers next period.

supply).[18] With this additional currency now available to it, at each date $t \geq 1$ the bank's problem becomes:

Problem *B*2 *Choose* $\{e_t^{1f}, e_t^{2f}, e_t^{2s}, l_t^{ij}, \chi_t^i, \kappa_t^i, (i, j = 1, 2), \chi_t^3, q_t, k_t\}$ *to maximize* $\boldsymbol{U}_t^{\boldsymbol{B}}$, *eq.* (1), *subject to eqs.* (2), (3), (5) - (9), *and*

$$(\lambda^2 - \lambda^1)e_t^{2s} = [\chi_t^2 + \chi_t^3]R_t^2 q_t + \kappa_t^2 x k_t, \tag{17}$$

$$0 \leq \chi_t^3 \leq \bar{\chi}_t^3. \tag{18}$$

Since the bank takes rates of return as given, inspection of eqs. (17) and (18) implies that the bank will choose the maximum transfer that the central bank will allow. Assuming that $\bar{\chi}_t^3$ is not so large as to drive currency out of circulation (which implies that $R^2 > x$; see fn. 18 below), then the optimal stationary bank contract again sets $\chi^1 = 1$ and $\chi^2 = \kappa^1 = 0$. These results obtain for the same reasons as in the previous section, namely that under stationarity market clearing implies that $R^1 = 1$ (see eq. [13]), so that rate of return dominance and the inability of the bank to determine the measure of early consumers implies that it pay reserves to as many middle-aged agents as possible and pay immature capital to as few such agents as possible. Although it is no longer necessary for the bank to choose a strictly positive value of κ^2 because the withdrawals of late-arriving early consumers may now be financed using currency, as shown below the bank will nevertheless again find it optimal to do so. As for the remaining parts of the bank's optimal contract, they depend upon the value of R^2 which in turn depends upon the value of $\bar{\chi}^3$. To determine this value requires consideration of the central bank's problem and the associated stationary equilibrium.

The central bank is assumed to solve the same problem as does the commercial bank with the exception that the central bank chooses $\bar{\chi}_t^3$ rather than χ_t^3 (so that eq. [18] no longer applies) and it internalizes the effects that its choice of $\bar{\chi}_t^3$ has on rates of return through the economy's market clearing conditions. It follows that the central bank's problem may be written as

Problem *CB* *Choose* $\{e_t^{1f}, e_t^{2f}, e_t^{2s}, l_t^{ij}, \chi_t^i, \kappa_t^i, (i, j = 1, 2), \bar{\chi}_t^3, q_t, k_t\}$ *to maximize* $\boldsymbol{U}_t^{\boldsymbol{B}}$, *eq.* (1), *subject to eqs.* (2), (3), (5) - (9),

$$(\lambda^2 - \lambda^1)e_t^{2s} = [\chi_t^2 + \bar{\chi}_t^3]R_t^2 q_t + \kappa_t^2 x k_t \tag{19}$$

and market clearing conditions in the presence of central bank lending, eq. (10) *for* $t = 1$; *eq.* (11) *and*

$$q_2 = [\chi_2^1 + \chi_2^2 + \bar{\chi}_2^3]R_1^2 q_1, \tag{20}$$

for $t = 2$; *and for* $t \geq 3$ *eqs.* (13), (14),

$$q_t = (\chi_{t-1}^1 + \chi_{t-1}^2 + \bar{\chi}_{t-1}^3)R_{t-1}^2 q_{t-1} + (1 - \chi_{t-2}^1)R_{t-2}^1 R_{t-1}^2 q_{t-2}, \tag{21}$$

$$q_t = (\chi_{t-1}^1 + \chi_{t-1}^2 + \bar{\chi}_{t-1}^3)R_{t-1}^2 q_{t-1} + (1 - \chi_{t-2}^1 - \chi_{t-2}^2)R_{t-2}^2 R_{t-1}^2 q_{t-2} \tag{22}$$

where the assumption that the additional currency reverts to the central bank at the end of the period implies

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[18] The time t+1 loan in real terms equals $p_{t+1}\Delta M$. Multiplying and dividing by $p_t M$ implies that $p_{t+1}\Delta M = R_t^2 q_t \Delta M/M$. It then follows that $\chi_t^3 = \Delta M/M$.

that it does not appear in the terms representing the supply of real balances of old agents.

In order to simplify the analysis and to be consistent with what appears above, I henceforth restrict matters to stationary monetary equilibria that are solutions to Problem *CB*, and, because the commercial bank is a price taker and so sets $\chi_t^3 = \overline{\chi}_t^3$, are also solutions to Problem *B*2. Denoting any such equilibrium as a SME^{CB}, the definition of same follows:

Definition 2 *Given $M > 0$ and the stochastic process $\{\tilde{\lambda}_t\}_{t=1}^{\infty}$, a* SME^{CB} *consists of scalars which are strictly positive, (p_1, R^1, R^2); nonnegative, $(e^{1f}, e^{2f}, e^{2s}, l^{11}, l^{12}, l^{21}, l^{22}, \overline{\chi}^3)$; and bounded between zero and one, $(q, k, \chi^1, \chi^2, \kappa^1, \kappa^2)$; such that* Problem *CB is solved for every $t \geq 1$.*

Assuming logarithmic utility, I now show that under certain conditions, there exists a SME^{CB}.

Proposition 2 *If $x \geq X^{-1}$, π is sufficiently close to zero, and λ^2 is sufficiently close to one, then there exists a unique* SME^{CB}.

As in a SME, the market clearing conditions in a SME^{CB} uniquely determine the stationary equilibrium values of the rates of return and two of the four share parameters. In particular, the stationary version of eq. (13) implies that $R^1 = 1$ while the stationary version of eq. (14) implies that $\chi^2 = 0$ and therefore that $\chi^1 = 1$. The stationary version of either eq. (21) or (22) then implies that $R^2 = (1 + \overline{\chi}^3)^{-1}$.[19] Furthermore, it is again the case that the uniform arrival of depositors and rate of return dominance imply that $\kappa^1 = 0$. Given these results, the remaining first-order conditions determine a system in q, κ^2, and $\overline{\chi}^3$ which is shown to have a unique solution when λ^2 is sufficiently large. The remaining two restrictions guarantee that $\chi^1 = 1$ and $\chi^2 = 0$ are indeed optimal for the central bank.

Through its elastic currency provision, a SME^{CB} relaxes the constraint on issuing currency to late-arriving middle-aged agents. When the loan is of the optimal size, it implies that $e^{2f} = e^{2s}$ so that risk sharing is now borne by all middle-aged agents when $\tilde{\lambda}_t = \lambda^2$ rather than by just those who arrive late. Consequently, a SME^{CB} supports an allocation that Pareto dominates the one that a SME supports. Moreover, since the solution to Problem *B*2 sets $\chi^3 = \overline{\chi}^3$ and the relevant market clearing conditions are now given by eqs. (13), (14), (21), and (22), it follows that in equilibrium the solution to Problem *B*2 is equivalent to that of Problem *CB* and hence yields the same Pareto-improving allocation.

It remains to determine whether the SME^{CB} supports an allocation that is also the solution to some form of planning problem. If so, then no further welfare gain is possible beyond that attributable to having an elastic currency. Given the economy's overlapping generations structure and the focus on stationary equilibria, a natural planning problem to consider is one that chooses the stationary allocation that maximizes the steady-state ex ante utility of all agents born at dates $t \geq 1$ subject to the economy's steady-state feasibility constraints. I denote the solution to this problem as the "golden rule" allocation since it represents this economy's analog to what Freeman (1996) refers to as "golden rule" allocations in his model.

Unlike the central bank, the planner is assumed to be able to observe the realization of $\tilde{\lambda}_t$. While this suggests that the planner can make the portfolio of generation t contingent upon this value, in fact this is not the case. If $\tilde{\lambda}_t$ is not observed until the beginning of time t+1, then this is trivially true. If $\tilde{\lambda}_t$ is observed at the beginning of time t, then to make agents' portfolios contingent upon this value the planner must have access to a one-period technology with a rate of return equal to at least one. This is so because the planner can achieve a return of one using an intergenerational transfer whereby the part of the endowment of generation t young agents is transferred to middle-aged agents who wish to consume in period t.[20] However, because no such technology exists, the planner has no option other than to utilize the intergenerational

[19] This expression implies that $\overline{\chi}^3$ cannot exceed $1/x - 1$ since at this point $R^2 = x$ and any further increase will drive currency out of circulation should withdrawal demand be large.

[20] It is not optimal to use the transfer to finance the consumption of old agents since they always do better to receive mature capital.

transfer since to do otherwise implies a one-period return of x from immature capital. Of course the same is true even if $\tilde{\lambda}_t$ is not observed until time t+1. In either case, since the realization of $\tilde{\lambda}_t$ is known by the time agents of generation t are to consume, the planner is able to treat all middle-aged agents the same. By the same token, the planner is also able to treat all old agents the same no matter the realizations of $\tilde{\lambda}_{t-1}$ and $\tilde{\lambda}_t$. Hence, $e^{1f} = e^1$, $e^{2f} = e^{2s} = e^2$, $l^{11} = l^{12} = l^1$, and $l^{21} = l^{22} = l^2$.

Let z be the per capita quantity of generation t's endowment that is transferred to the middle-aged agents of generation t-1. Then the golden rule allocation is the solution to the following problem:

Problem *GR* *Choose* $\{e^1, e^2, l^1, l^2, \kappa^1, \kappa^2, z, k\}$ *to maximize*

$$\boldsymbol{U}^{\boldsymbol{GR}} = (1-\pi)\left[\lambda^1 u(e^1) + (1-\lambda^1)u(l^1)\right] + \pi\left[\lambda^2 u(e^2) + (1-\lambda^2)u(l^2)\right] \tag{23}$$

subject to

$$\lambda^1 e^1 = z + \kappa^1 xk, \tag{24}$$

$$\lambda^2 e^2 = z + \kappa^2 xk, \tag{25}$$

$$(1 - \lambda^1)l^1 = (1 - \kappa^1)Xk, \tag{26}$$

$$(1 - \lambda^2)l^2 = (1 - \kappa^2)Xk, \tag{27}$$

$$z + k = 1, \tag{28}$$

$$0 \le \kappa^i \le 1, \quad i = 1, 2 \tag{29}$$

where the κ^i are (effectively) the same as in Problems *B*1, *B*2, and *CB*.

I am now able to answer the question posed above, namely does a SME^{CB}, and hence a SME with elastic currency, support a golden rule allocation. The following proposition shows the answer to be yes.

Proposition 3 *A* SME^{CB} *supports a golden rule allocation.*

To prove the proposition, it suffices to show that the constraints and first-order conditions of a SME^{CB} are the same as those that solve Problem *GR*. Since in the equilibrium $\chi^1 = 1$, $\chi^2 = \kappa^1 = 0$, and the optimal central bank loan serves to eliminate completely the partial suspension of withdrawals when withdrawal demand is high, the proposition follows directly.

6. Conclusion

This paper offers an example of a monetary banking model in which the bank faces aggregate risk governing its expected withdrawals. In the model, the bank's best strategy is to hold both a fiat reserve asset and an illiquid asset, capital, against its deposits and to impose a partial suspension of withdrawals on certain agents once it learns that withdrawal demand exceeds a particular limit. Prior to reaching this limit, the bank finances the withdrawals of middle-aged agents entirely with reserves. Once this limit is reached, the withdrawals of the remaining middle-aged agents are financed entirely with immature capital. In contrast, the withdrawals of all old agents are financed entirely with mature capital.

Given that the bank would prefer to finance (in part) the withdrawals of late-arriving middle-aged agents with fiat currency, a mechanism that relaxes this constraint can be welfare improving. The one such mechanism that this paper considers is for the central bank to operate a type of discount window whereby it lends the commercial bank additional units of currency that are partially collateralized with (claims on) immature capital. Assuming that the central bank optimally chooses the maximum loan size, then the resulting stationary monetary equilibrium Pareto dominates the one without the loan since the bank is now better able to spread the risk of high withdrawal demand across early consumers. Moreover, the stationary equilibrium with central bank lending supports the "golden rule" allocation and so achieves the highest level of welfare consistent with maximizing steady-state ex ante expected utility.

Colophon

Some of the work on this paper, which previously circulated under the title "Demand Deposit Banking with a Fiat Reserve Asset and Intraday Credit," was undertaken while the author served as a Visiting Scholar at the Federal Reserve Bank of Atlanta. I thank the members of the Atlanta Fed Research Department, and in particular Will Roberds, for their helpful discussions. The views expressed herein, however, are not necessarily those of the Federal Reserve Bank of Atlanta nor of Federal Reserve System. I thank as well the editor, Per Krusell, two referees, and participants at the 2001 Midwest Macroeconomics Conference for their comments and suggestions. All remaining errors are solely of my own creation.

Appendix

Proof of Proposition 1

Given logarithmic utility and the mutual-like structure of the bank, I claim that $\{e^{1f}, e^{2f}, e^{2s}, l^{11}, l^{12}, l^{21}, l^{22}, q, k, \chi^1, \chi^2, \kappa^1, \kappa^2, p_1, R^1, R^2\} = \{1, 1, x, X, X, X, X, \lambda^1, 1 - \lambda^1, 1, 0, 0, (\lambda^2 - \lambda^1)/(1 - \lambda^1), M/\lambda^1, 1, 1\}$ is the unique stationary monetary equilibrium if and only if $1 \geq \pi/x + (1 - \pi)/X$.

if Inspection of eqs. (10) - (16) shows that the conjectured equilibrium satisfies market clearing at every date. Likewise, inspection of eqs. (2) - (8) shows that the conjectured equilibrium satisfies the budget constraints of Problem $B1$ at every date. Thus, in the sequel, the notation may be simplified to read $e_t^{1f} = e_t^{2f} = e^f$, $l_t^{11} = l_t^{12} = l^1$, $l_t^{21} = l_t^{22} = l^2$, and for consistency, $e_t^{2s} = e^s$.

Consider next Problem $B1$. Given $R^1 = R^2 = 1$, this problem may now be written more succinctly as choosing $\{e^f, e^s, l^1, l^2, \chi^1, \chi^2, \kappa^1, \kappa^2, q, k\}$ to maximize

$$\boldsymbol{U^B} = \lambda^1 u(e^f) + (1-\pi)(1-\lambda^1)u(l^1) + \pi\left[(\lambda^2-\lambda^1)u(e^s) + (1-\lambda^2)u(l^2)\right] \tag{A1}$$

subject to

$$\lambda^1 e^f = \chi^1 q + \kappa^1 x(1-q), \tag{A2}$$

$$(\lambda^2-\lambda^1)e^s = \chi^2 q + \kappa^2 x(1-q), \tag{A3}$$

$$(1-\lambda^1)l^1 = (1-\chi^1)q + (1-\kappa^1)X(1-q), \tag{A4}$$

$$(1-\lambda^2)l^2 = (1-\chi^1-\chi^2)q + (1-\kappa^1-\kappa^2)X(1-q). \tag{A5}$$

Since the self-selection constraint is nonbinding in equilibrium (Diamond and Dybvig, 1983), it follows that the first-order conditions for q, χ^1, χ^2, κ^1, and κ^2 must satisfy (after substituting out multipliers for marginal utilities):

$$u'(e^{\mathrm{f}}) - X[(1-\pi)u'(l^1) + \pi u'(l^2)] = 0, \tag{A6}$$

$$\{u'(e^{\mathrm{f}}) - [(1-\pi)u'(l^1) + \pi u'(l^2)]\}q - \nu_1 = 0, \quad \nu_1(1-\chi^1-\chi^2) = 0, \tag{A7}$$

$$\{\pi[u'(e^{\mathrm{s}}) - u'(l^2)]\}q - \nu_1 + \nu_2 = 0, \quad \nu_2\chi^2 = 0, \tag{A8}$$

$$\{xu'(e^{\mathrm{f}}) - X[(1-\pi)u'(l^1) + \pi u'(l^2)]\}(1-q) + \nu_3 = 0, \quad \nu_3\kappa^1 = 0, \tag{A9}$$

$$xu'(e^{\mathrm{s}}) - Xu'(l^2) = 0, \tag{A10}$$

for $\nu^1, \nu^2, \nu^3 \geq 0$.

Let $u = \ln(c)$. Substitution of the conjectured equilibrium values into eqs. (A6) - (A10) shows that all but (A8) hold unconditionally. As for eq. (A8), elimination of ν_1 using (A7) shows that the former may now be written as

$$u'(e^{\mathrm{f}}) \geq \pi u'(e^{\mathrm{s}}) + (1-\pi)u'(l^1). \tag{A11}$$

Given the conjectured equilibrium, eq. (A11) holds if $1 \geq \pi/x + (1 - \pi)/X$. Uniqueness follows from the concavity of the utility function and the convexity of the budget set. ■

only if Let $u = \ln(c)$ and suppose that the conjectured values constitute a stationary monetary equilibrium so that these values solve Problem $B1$ and market clearing at every date. Hence, they satisfy, among other things, eqs. (A7) and (A8). Substituting (A7) into (A8) to eliminate ν_1 and then applying the conjectured equilibrium values implies that $1 \geq \pi/x + (1 - \pi)/X$. ■

Proof of Proposition 2

Consider first the implications of market clearing. The stationary version of eq. (13) implies that $R^1 = 1$ while the stationary version of eq. (14) implies that $\chi^2 = 0$ and therefore that $\chi^1 = 1$. The stationary version of either eq. (21) or (22) then implies that $R^2 = (1 + \overline{\chi}^3)^{-1}$.

Consider next optimization. Suppose that R^1, R^2, χ^1 and χ^2 equal their equilibrium values. Since these imply that $l^{11} = l^{12} = l^1$ and $l^{21} = l^{22} = l^2$, Problem CB consists of choosing $\{e^{1\mathrm{f}}, e^{2\mathrm{f}}, e^{2\mathrm{s}}, l^1, l^2, \chi^1, \chi^2, \overline{\chi}^3, \kappa^1, \kappa^2, q, k\}$ to maximize

$$\boldsymbol{U^{CB}} = (1-\pi)\left[\lambda^1 u(e^{1\mathrm{f}}) + (1-\lambda^1)u(l^1)\right] + \pi\left[\lambda^1 u(e^{2\mathrm{f}}) + (\lambda^2-\lambda^1)u(e^{2\mathrm{s}}) + (1-\lambda^2)u(l^2)\right] \tag{A12}$$

subject to

$$\lambda^1 e^{1f} = \chi^1 q + \kappa^1 x k, \tag{A13}$$

$$\lambda^1 e^{2f} = \chi^1 (1 + \overline{\chi}^3)^{-1} q + \kappa^1 x k, \tag{A14}$$

$$(\lambda^2 - \lambda^1) e^{2s} = (\chi^2 + \overline{\chi}^3)(1 + \overline{\chi}^3)^{-1} q + \kappa^2 x k, \tag{A15}$$

and eqs. (A4) and (A5). The first-order conditions for q, χ^1, χ^2, $\overline{\chi}^3$, κ^1 and κ^2 may then be written as

$$(1 - \pi) u'(e^{1f}) + \pi u'(e^{2s}) = X[(1 - \pi) u'(l^1) + \pi u'(l^2)] \tag{A16}$$

$$\{(1 - \pi) u'(e^{1f}) + \pi u'(e^{2f})\cdot(1 + \overline{\chi}^3)^{-1} - [(1 - \pi) u'(l^1) + \pi u'(l^2)\cdot(1 + \overline{\chi}^3)^{-1}]\cdot\Pi\} q - \nu_1 = 0,$$
$$\nu_1(1 - \chi^1 - \chi^2) = 0, \tag{A17}$$

$$\{u'(e^{2s}) - u'(l^2)\cdot\Pi\}\pi(1 + \overline{\chi}^3)^{-1} q - \nu_1 + \nu_2 = 0, \qquad \nu_2 \chi^2 = 0, \tag{A18}$$

$$[x u'(e^{1f}) - X u'(l^1)](1 - \pi)(1 - q) + \nu_3 = 0, \qquad \nu_3 \kappa^1 = 0, \tag{A19}$$

$$e^{2f} = e^{2s}, \tag{A20}$$

$$x u'(e^{2s}) = X u'(l^2), \tag{A21}$$

where $\Pi = 1 - \pi + \pi(1 + \overline{\chi}^3)^{-1}$. Substituting the relevant budget constraints into eqs. (A16), (A20), and (A21) yields a system in $(q, \kappa^2, \overline{\chi}^3)$. After some algebra, this system reduces to

$$\frac{(1 - \lambda^2)}{x(\lambda^2 - \kappa^2)} = \frac{(1 - \pi)(1 - \lambda^1)(1 - \kappa^2) + \pi(1 - \lambda^2)}{(1 - \pi)\lambda^1(1 - \kappa^2) + \pi(\lambda^2 - \kappa^2)}. \tag{A22}$$

Let $L(\kappa^2)$ and $R(\kappa^2)$ denote the left- and right-hand sides of (A22). It follows that $\lim_{\kappa^2 \to \lambda^2} L(\kappa^2) = \infty$ and $\lim_{\kappa^2 \to \overline{\kappa}^2} R(\kappa^2) = \infty$ where $\overline{\kappa}^2 = \dfrac{(1 - \pi)\lambda^1 + \pi\lambda^2}{(1 - \pi)\lambda^1 + \pi}$. Since $\lambda^2 < \overline{\kappa}^2$ and $L(0) = \dfrac{1 - \lambda^2}{x\lambda^2}$, it follows that if λ^2 is sufficiently close to one, then $L(0) < R(0)$ and so there exists a unique solution to eq. (A22). Given this solution, the optimal solutions for $\overline{\chi}^3$ and q are given by $\overline{\chi}^3 = \dfrac{(\lambda^2 - \kappa^2)}{\lambda^1(1 - \kappa^2)} - 1$ and $q = \dfrac{x(\lambda^2 - \kappa^2)}{x(\lambda^2 - \kappa^2) + (1 - \lambda^2)}$.

It remains to verify that eqs. (A17) - (A19) hold. Using eqs. (A16) and (A21), it is readily seen that (A19) holds. As for eq. (A17), applying eq. (A16) implies that (A17) is satisfied for $\nu_1 \geq 0$ if

$$(1 - \pi) u'(e^{1f})(1 - X^{-1}) + \pi u'(e^{2s})[(1 + \overline{\chi}^3)^{-1} - X^{-1}] \geq 0. \tag{A23}$$

Since $(1 + \overline{\chi}^3)^{-1} \geq x$ is necessary for the existence of a monetary equilibrium, a sufficient condition for eq. (A23) to hold is $x \geq X^{-1}$. Turning to eq. (A18), eliminating ν_1 using eq. (A17) implies that (A18) is satisfied for $\nu_2 \geq 0$ if $e^{1f} \leq l^1$. Eq. (A16) then shows this condition is satisfied if π is sufficiently close to zero. ■

Proof of Proposition 3

Given that an SME^{CB} implies that $e^{2f} = e^{2s}$, $\chi^1 = 1$, and $\chi^2 = \kappa^1 = 0$, inspection of eqs. (A12) - (A15) plus eqs. (A4) and (A5) shows them to be equivalent to eqs. (23) - (29) with q playing the role of z and eqs. (A14) and (A15) implying eq. (25). Furthermore, direct computation shows the first-order conditions from Problem *GR* to be equivalent to eqs. (A16), (A19), and (A21) given the above set of equalities. ■

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