ECONOMIES OF SCALE IN MUNICIPAL POLICE DEPARTMENTS: THE CASE OF FLORIDA

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Abstract—A multiproduct translog cost function is estimated using cross section data from Florida's municipal police departments. The estimated coefficients are used to calculate economies of scale in police production. We find significant scale diseconomies in police production in our sample. The sources of these scale diseconomies are large municipal police departments. We also find no evidence of economies of scope.

I.

Arguments for consolidation of the provision of certain municipal services such as water, lighting, and police services are based on the presumed existence of substantial economies of scale in the production of such services. In the case of police services, if such economies do exist, then a strong economic case could be made for the consolidation of police departments in adjoining municipalities.

Estimates of scale economies in the provision of police services have yielded mixed results at best. Walzer (1972) and Chapman, Hirsch, and Sonenblum (1975) find police agencies enjoying increasing returns to scale. Ehrlich (1973), Popp and Sebold (1972), and Votey and Philips (1972), using disaggregate police outputs, find decreasing returns to scale in all police outputs.

Two characteristics are common to the studies cited. First is the use of the Cobb-Douglas functional form. This functional form puts severe restrictions on the structure of the underlying production technology. In particular, the implied constant elasticity of substitution and homotheticity of the production function may not be justified. At best, these assumptions should be maintained as testable hypotheses.

Second, these studies treat the police department as producing a single output despite the fact that the police produce many services with the same inputs. Where the police department is recognized as a multiproduct "firm," these studies have assumed nonjointness in output, making it possible to estimate a separate production function for each output produced by the police. Darrough and Heineke (1978) have shown that police production is joint; hence estimating a separate production function for each output could lead to biased estimates.[1]

Darrough and Heineke (1978), using the multi-product translog cost function and data from police departments in medium-size cities in the 1970s, find neither increasing nor decreasing returns to scale in the average police department, even though they find wide variation in scale economies among police departments in their sample. While Darrough and Heineke overcome the problems associated with the earlier studies, they collapse all inputs into one—police personnel. The use of civilians for clerical and administrative jobs in police departments has become very common.[2] Yet Darrough and Heineke do not account for civilian employees in their cost estimates. The exclusion of other inputs could lead to biased parameter estimates.

This paper estimates scale economies in police production, using data from municipal police departments in the state of Florida. We estimate a multi-product translog cost function (Christensen, Jorgenson, and Lau, 1973) and apply duality theorems (Diewert, 1982) to derive economies of scale of the underlying production function. We also test for the existence of economies of scope. It is by estimating scale economies from different data sets under different conditions that the extent of scale economies in police production in general can be inferred.

The modelling follows Darrough and Heineke (1978, 1979), with two differences. First, we use a different data set with more observations to aid efficient estimation. Second, we disaggregate labor into sworn (police) and non-sworn (civilian) personnel. We also include a capital input.

The paper is organized as follows: Section II discusses the model; the data are discussed in section III while the statistical results are presented and discussed in section IV. Section V concludes the paper.

II.

Following Darrough and Heineke (1978), we assume that the police department's problem is to choose the level and mix of output to maximize the net value of

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[1] See Darrough and Heineke (1978) for further discussion of these models.

[2] In 1980, 27% of all employees in Florida municipal police departments were civilian. This proportion increased to 29% in 1984. The corresponding figures for the sheriffs departments were 36% and 48.3%, respectively.

police output. Formally the problem is set up as in equation (1) below.

$$\text{Max } P_Y \cdot Y - C(Y, W) \qquad (1)$$

where

P = vector of output prices,
Y = vector of outputs,
W = vector of input prices, and
$C(\cdot)$ = the joint cost function.

First-order conditions for value maximization of (1) implies that

$$P_i = C_{yi}, \qquad i = 1, \ldots, m \quad (2)$$

where C_{yi} is the marginal cost of producing Y_i. For efficient parameter estimates, equations (1) and (2) can be jointly estimated.

It is possible to estimate the above system only if there are prices for the outputs and inputs. For crimes against the person, it is not possible to attach prices to their solution. For property crimes, however, it is possible to think of the average value of stolen property as the price the police will attach to their solution. We therefore follow Darrough and Heineke in using average value of stolen goods as the price for the solution to property crimes and assume functional separability of property crimes solution and other activities of police departments.[3] Functional separability allows us to estimate the parameters of the solution to property crimes.

We approximate our cost function with the multiproduct translog cost function. The translog cost function is a polynomial approximation to any twice differentiable cost function and, as such, imposes few restrictions on the parameters of the underlying production function. In particular, it places no restrictions on returns to scale and elasticity of substitution between any pair of inputs. A cost function should, however, obey certain regularity conditions. The cost function is linearly homogeneous in input prices and the second order coefficients are symmetric.

The output measure we employ is total arrests of the seven FBI index crimes. We aggregate all personal crimes into one and represent all other activities of the police not related to arrest by the population of the community.[4] This gives six outputs. There are two forms of labor—police and civilian and a composite capital input. With six outputs and three inputs, the multi-product translog cost function with linear homogeneity and symmetry restrictions imposed can be written as

$$\begin{aligned}\ln C = \alpha_0 &+ \sum_{i=1}^{6} \alpha_i \ln Y_i + \sum_{j=1}^{3} \beta_j \ln W_j \\ &+ \frac{1}{2} \sum_{i=1}^{6} \sum_{j=1}^{6} \alpha_{ij} \ln Y_i \ln Y_j \\ &+ \frac{1}{2} \sum_{j=1}^{3} \sum_{k=1}^{3} \beta_{jk} \ln W_j \ln W_k \\ &+ \sum_{i=1}^{6} \sum_{j=1}^{3} \gamma_{ij} \ln Y_i \ln W_j \\ &\alpha_{ij} = 0 \text{ for } i = 1 \ldots 4,\ j = 5, 6 \qquad (3)\end{aligned}$$

where $Y_{i,j}$ = output i, j; $W_{j,k}$ = price of input j, k; and α_i, β_j, α_{ij}, β_{jk} and γ_{ij} are parameters.

Logarithmic differentiation of (3) with respect to output gives the output cost shares:

$$S_i = \alpha_i + \sum_j \alpha_{ij} \ln Y_j + \sum_j \gamma_{ij} \ln W_j, \qquad \text{for } i, j \text{ and } k \quad (4)$$

where $S_i = P_i Y_i/\text{Total Cost}$. Equations (3) and (4) form the system of equations to be estimated.

III.

Output and employment data were obtained from *Crime in Florida* (Florida Department of Law Enforcement [FDLE], 1982 and 1983). Average values of stolen goods were obtained from unpublished FDLE sources. Police wage, calculated as the unweighted average annual income of police officers in a jurisdiction, were obtained from the *Annual Reports* of the Florida Criminal Justice Standards and Training Commission (FCJST).

Neither the FDLE nor the FCJST provide data on the incomes of civilian employees. Since most civilians are employed in clerical and administrative positions, we used the average hourly wage of clerical workers in the jurisdiction as a proxy for civilian wages. These data came from Florida Department of Labor and Employment Security, *Job Openings Filled and Hourly Wages*: 1982, 1983. The hourly wage was converted to yearly equivalent. The civilian wage did not include fringe benefits, while the police wage did. To make the two wages comparable, we adjusted the civilian wages upwards by 20%.[5] Total expenditure data were obtained from *Local Government Expenditures* (Florida Department of Banking and Finance, 1982 and 1983).

[3] See Darrough and Heineke (1979).

[4] Though such services produced may not be quantifiable, they are likely to be proportional to the population in a jurisdiction. Such a measure has also been used by other researchers. See Darrough and Heineke (1978).

[5] The 20% figure was suggested to the author in a telephone conversation with the Labor Department's Southeastern Office in Atlanta on July 31, 1985.

We used the price of a fully equipped Oldsmobile Delta 88 as our proxy for the price of capital.[6] All nominal variables were deflated with two price indexes —first, Florida's annual comparative price index to remove spatial price difference,[7] and second, the CPI for the southeastern United States to adjust for temporal price changes.

The data are for police departments of municipalities in the state of Florida with a population of 5000 or more for the years 1982 and 1983. Of the 153 municipalities that met this criterion, not all police departments reported in all years. There were a total of 256 observations over the two-year period. All variables were mean scaled so the logarithm of a variable at its mean is zero.

IV.

Since the data used consisted of two cross section samples, we tested to see if the data could be pooled. We could not reject the null hypothesis that parameters from the two years are the same.[8]

Kmenta and Gilbert (1968) have shown that the Iterative Zellner estimation procedure produces asymptotically maximum likelihood estimates. Furthermore, parameter estimates do not depend upon which share equation is deleted from the system.[9] Given these characteristics, we used the Iterative Zellner efficient estimation method (Zellner, 1962) to estimate the parameters of the system (3) and (4).

The corrected R^2s for the equations are 0.07, 0.17, 0.26, 0.32 and 0.956 for *ROB*, *BURG*, *LARC*, and *MVT* share equations and the cost equation, respectively.[10] The estimated coefficients are reasonable. With the exception of the coefficient on the price of capital, β_3, all the first order as well as direct second order input price coefficients were significantly positive. The negative coefficient on the price of capital is insignificant. All outputs enter the cost equation with positive coefficients that are significantly different from zero at any reasonable degree of confidence. For all property crimes, the direct second order output coefficients are significantly positive, implying that cost of producing that output, after a certain level of output has been attained, may increase at an increasing rate. This may be consistent with a U-shaped average cost curve.

Our strategy for measuring and testing economies of scale is as follows: We first impose constant returns to scale restriction on the underlying police production function. If constant returns to scale is rejected, we proceed to measure scale economies in the "average" police department. The sample is then partitioned into small, medium, and large city groups and cost equations estimated for each subgroup. We then calculate scale economies from the parameters of the estimated cost function. We do this to find out if there are any efficiency differences in police departments of various sizes.

In their 1978 paper, Darrough and Heineke used only one input price—police wages—to estimate their cost function from which they calculated scale economies. We estimate such an equation from our data, calculate the implied scale economies and compare it to our more expanded model. Early research on police production employed the Cobb-Douglas production function. We test to find out if the underlying production function is of the homogeneous Cobb-Douglas type.

Our test of all hypotheses is based on the statistic $-2 \log \Lambda = N(\log|\Omega_r| - \log|\Omega_u|)$; where $|\Omega_r|$ and $|\Omega_u|$ are the absolute values of the determinant of the error covariance matrices for restricted and unrestricted models respectively, and N is the number of observations. This statistic is distributed as chi-square, with degrees of freedom equal to the number of parameter restrictions imposed. This test is a likelihood ratio test. The calculated test statistics, degrees of freedom, and critical values are presented in table 1.

The calculated statistic for the null hypothesis of constant returns to scale is 51.04. With 9 degrees of freedom, this hypothesis is soundly rejected. The test statistics for homogeneous Cobb-Douglas and police as the only input hypotheses are 124.60 and 383.32, respectively. Both hypotheses are easily rejected at any reasonable level of confidence. This casts doubts on the appropriateness of the use of the Cobb-Douglas function to model police production.

Christensen and Green (1976) have defined and measured economies of scale (*SCE*) as unity minus the proportionate change in cost when output is changed by a certain quantity:

$$SCE = 1 - \partial \ln C / \partial \ln Y. \tag{5}$$

If *SCE* is positive, there are increasing returns to scale; negative values imply decreasing returns to scale while a value of zero implies constant returns to scale. For a multi-product cost function *SCE* is defined as unity minus the change in cost resulting from a proportionate

[6] Philips (1978) has used such a measure in his study of the demand for inputs in police production. The Sarasota city police department also confirmed to the author that the model of automobile used here is a reasonable approximation of police vehicles in Florida.

[7] Florida's cost of living index is calculated for counties and SMSAs only. Each jurisdiction's nominal variables were deflated with the cost of living index of the county in which it is located. We were therefore able to adjust for inter-county cost of living differentials but not for intra-county differentials.

[8] The calculated statistics were $F_{37,182} = .882$.

[9] We did not have to delete any share equations because the assumption of separability allowed us to estimate shares for only property crimes.

[10] Detailed parameter estimates are available upon request from the author.

TABLE 1.—STATISTICS FOR TESTING HYPOTHESES ABOUT THE PRODUCTION STRUCTURE

Hypothesis	Test Statistic	Degrees of Freedom	Critical Value[a]
1. Constant Returns to Scale	51.04	9	21.7
2. Cobb-Douglas	124.60	27	46.96
3. Police as the only input	383.32	16	32.0
4. No Economies of Scope:			
i. *ROB* and *BURG*: $\alpha_1\alpha_2 + \alpha_{12} = 0$	1.36	1	6.63
ii. *ROB* and *LARC*: $\alpha_1\alpha_3 + \alpha_{13} = 0$	0.18	1	6.63
iii. *ROB* and *MVT*: $\alpha_1\alpha_4 + \alpha_{14} = 0$	1.01	1	6.63
iv. *BURG* and *LARC*: $\alpha_2\alpha_3 + \alpha_{23} = 0$	1.03	1	6.63
v. *BURG* and *MVT*: $\alpha_2\alpha_4 + \alpha_{24} = 0$	1.87	1	6.63
vi. *LARC* and *MVT*: $\alpha_3\alpha_4 + \alpha_{34} = 0$	1.48	1	6.63
vii. *PERS* and *POP*: $\alpha_5\alpha_6 + \alpha_{56} = 0$	1.00	1	6.63

[a]All tests at $\alpha = .01$.

change in all outputs.

$$SCE = 1 - \sum_i \partial \ln C / \partial \ln Y_i. \qquad (5')$$

For the multi-product translog cost function, this implies that

$$SCE = 1 - \left(\sum_i \alpha_i + \sum_i \sum_j \alpha_{ij} \ln Y_j + \sum_i \sum_j \gamma_{ij} \ln W_j \right). \qquad (5'')$$

Since we mean scale our data, the last two expressions in parentheses vanish, reducing the measure of *SCE* to be $1 - \Sigma_i \alpha_i$. $\Sigma_i \alpha_i$ is calculated from table 1 to be equal to 1.0688 implying *SCE* of $-.0688$.

On the surface, *SCE* of $-.0688$ implies a relatively small decreasing returns to scale. However, the question is whether this *SCE* measure is significantly different from zero. The calculated *SCE* is significantly different from zero at $\alpha = .05$. The conclusion from this result is that there are diseconomies of scale in the "average" police department in Florida.

This result pertains to the "average" police department; there could be variation in scale economies among police departments in the sample. To investigate the source of the observed scale diseconomies, we divided our sample into large, medium, and small police departments, estimated the cost function for each, and calculated scale economies for each subsample. Large departments are those serving cities with population of 50,000 or more; medium departments serve cities with population between 25,000 and 50,000, while small departments serve cities with population of less than 25,000.

The calculated scale economies are presented in table 2. From this table, we see that production in small and large police departments is characterized by decreasing returns to scale. The calculated negative *SCE* for small cities is, however, statistically insignificant. Police production in medium-size cities is characterized by increasing returns to scale even though the calculated *SCE* is statistically insignificant. One may conclude that the major source of scale diseconomies in our sample is the large cities.

Does the exclusion of other inputs affect the estimates of scale economies? The calculated *SCE* associated with the restricted model is 0.1468 as shown in table 2.

TABLE 2.—ESTIMATES OF SCALE ECONOMIES

Jurisdiction	*SCE* Estimate	Standard Error
All Cities: (256)		
a. Full Model	−.0688	.0343
b. With only police input	.1468	.1356
Large Cities (41)	−.3833	.1542
Medium-Size Cities (50)	.1995	.3052
Small-Size Cities (165)	−.0155	.2865

Note: Number of observations in parentheses.

Apparently, the exclusion of capital and other inputs from the cost function shows up as a reduction in the aggregate output elasticity of cost. We note, however, that the estimated positive returns to scale is statistically insignificant, a result that is similar to that of Darrough and Heineke (1978).

The conclusion from this study is consistent with the scale diseconomies obtained by Ehrlich (1973), Popp and Sebold (1972), and Votey and Philips (1972). It is inconsistent with the scale economies obtained by Chapman, Hirsch, and Sonenblum (1975) and Walzer (1972). We obtain the Darrough and Heineke (1978) results only when we exclude the price of capital and labor inputs.

Though police production in Florida exhibits decreasing returns to scale, it is possible that production is characterized by economies of scope (EOS) because of interproduct complementarity. EOS exists if the following condition holds:

$$C(Y_i, Y_j) < \left[C(Y_i, 0) + C(0, Y_j)\right]. \quad (6)$$

A test for this condition to exist is

$$\alpha_i \alpha_j + \alpha_{ij} < 0 \quad (7)$$

where output and price variables have been normalized around their means and α's are estimated output coefficients.[11]

We test for the existence of EOS in our sample. Test statistics for the hypotheses of no EOS are shown in the bottom part of table 1. We are unable to reject any of the seven hypotheses at any reasonable level of confidence. There is no statistically significant EOS.

Conclusion

The objective of this paper was to measure the extent of scale economies in municipal police departments in the state of Florida. Using the multi-product translog cost function, we found significant diseconomies of scale in the average police department in Florida. Analysis of scale economies by city size showed that large cities were the major source of the scale diseconomies observed in our sample. We also could not find evidence of economies of scope. In spite of any shortcomings, this conclusion and the results from previous research should sound a note of caution to the proponents of consolidation of municipal police departments on account of substantial scale economies.

[11] See Denny and Pinto (1978) and Murray and White (1983).

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