

## ***Information-Based Bank Runs in a Monetary Economy\****

This paper offers a general equilibrium model of banking that is capable of confronting a number of “stylized facts” from the 1929–1933 period of U.S. banking history. In the model, fiat currency is part of the bank’s portfolio, banks are subject to an explicit sequential service constraint, and bank runs are information-based. After describing the bank’s problem and defining an equilibrium, a simulation of the economy’s equilibrium is provided. Key features of the simulation are shown to be consistent with the “stylized facts.”

“To understand the Great Depression is the Holy Grail of macroeconomics”  
(Bernanke 1995, 1).

### **1. Introduction**

Explaining the 1929–1933 period in U.S. banking history represents an important challenge for general equilibrium banking models. According to evidence reported by Friedman and Schwartz (1963), Cagan (1965), Bernanke (1983), and others, such models must be capable of delivering an equilibrium in which a banking crisis yields such features as a partial suspension of withdrawals, the calling of potentially productive loans in order to assist in the financing of withdrawals, and a contemporaneous decline in the price level despite the presence of a relatively stable monetary base. Furthermore, the magnitude of the decline in the price level should increase with the magnitude of the crisis.

In order that a model of banking be able to confront these “stylized facts” of the 1929–1933 period, it is necessary for the model to include fiat currency. Although this conclusion seems fairly obvious, there nevertheless exist few examples of banking models that explicitly include currency.<sup>1</sup> In

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<sup>1</sup>Among those that do are Bryant (1980), Smith (1987), Loewy (1991, 1995), and Champ, Smith, and Williamson (1996). For comprehensive reviews of the literature on banking and liquidity see Jacklin (1989), Dowd (1992), and Bhattacharya and Thakor (1993).

this paper, I extend a version of the “information-based” bank run model of Jacklin and Bhattacharya (1988) and Chari and Jagannathan (1988) to a three-period-lived overlapping generations model of money.<sup>2</sup> To this environment I add the assumption, due to Wallace (1988, 1990), that agents begin their second period of life isolated from each another. This assumption, which produces a version of Diamond and Dybvig’s (1983) sequential service constraint, implies that the bank cannot determine the measure of agents seeking to withdraw until it has made payments to at least some agents.

As documented below through a simulation of the economy’s equilibrium, the model of this paper successfully confronts the evidence described above. Indeed, the simulated equilibrium is such that if a run occurs, then the bank finds it optimal to use both currency and called loans to finance withdrawals, and to partially and then fully suspend withdrawals to those who are “last in line” to withdraw. Since certain agents who withdraw during a run save the proceeds of their withdrawal, the resultant fluctuation in the market for fiat currency causes a concurrent decline in the price level, one which worsens as the size of the run increases.

Modeling the macroeconomic effects of banks runs has also been the subject of studies by Waldo (1985), Loewy (1991), and Champ, Smith, and Williamson (1996). Waldo considers a three-period model in which agents are subject to an unknown “liquidity assessment” and the presence of costs of entry imply that assets other than currency can only be acquired through a bank. In this set-up, a bank run (*i.e.*, a period with a large liquidity assessment) causes (among other things) an increase in the currency-deposit ratio. However, because the demand for real balances is effectively zero in Waldo’s model in the absence of a run, it is difficult to interpret this result. Since the equilibrium of this paper is such that the demand for real balances is strictly positive in all states, such a difficulty does not arise.

Loewy (1991) describes a version of Diamond and Dybvig’s (1983) model in which real balances are subject to a reserve requirement. Thus, he is able to make meaningful comparisons of variables such as the price level and the deposit-currency ratio across the economy’s run and no run equilibria. While these comparisons yield results which are consistent with the evidence, Loewy effectively assumes that a run occurs with probability zero and so does not get incorporated into the bank’s problem. In contrast, the present paper assumes that a run occurs with positive probability. This implies that the effects of a run are included in the bank’s problem.

Champ, Smith, and Williamson (1996) study a dynamic general equi-

<sup>2</sup>Bernanke and Gertler (1987), Smith (1987), Freeman (1988), Loewy (1991, 1995), and Qi (1994) also develop banking models in an overlapping generations framework.

librium economy in which agents are subject to a location shock rather than to the usual preference shock. Currency enters their model by serving as the only transportable asset available to agents who need to relocate. However, because Champ, Smith, and Williamson (1996) are particularly interested in modeling events which took place under the National Banking System, their model is not specifically designed to describe the experience of the early 1930s. In addition, since they do not model sequential service, the bank in their model does not face the type of aggregate risk which is prominent in the economy of this paper.

The next section first describes the economy and then the implications of the isolation assumption. The bank's problem is the subject of Section 3. Section 4 defines an equilibrium while Section 5 provides a simulated equilibrium and shows how it is consistent with the "stylized facts" denoted above. Section 6 concludes.

## **2. The Model**

### *Economic Fundamentals*

Consider an overlapping generations economy where, at each date  $t \geq 1$ , a continuum (taken to be of measure one) of ex ante identical, three-period-lived agents is born. The generation to which each agent belongs is public information. Each agent born at time  $t$  is endowed with one unit of the economy's lone consumption good during time  $t$  and no units at any other date. As in Diamond and Dybvig (1983) and elsewhere, the members of generation  $t$  are of two types, measure  $\lambda_1$  ( $0 < \lambda_1 < 1$ ) who consume during their second period of life (early consumers) and measure  $1 - \lambda_1$  who consume during their third period of life (late consumers). While  $\lambda_1$  is assumed to be nonstochastic and public information, an agent's type is private information that does not become revealed to the agent until the beginning of time  $t + 1$ . Last, there does not exist a technology that permits an agent's type to be verified by other agents or by an intermediary.

Let  $e(t)$  and  $l(t)$  be time  $t + 1$  (early) and time  $t + 2$  (late) consumption of an agent born at time  $t \geq 1$ . Since an agent's type determines when he consumes, it also determines his utility. Thus, each agent's utility function is given by  $U[e(t), l(t), \theta] = \theta u[e(t)] + (1 - \theta)u[l(t)]$ , where  $\theta = 0$  if the agent is a late consumer and  $\theta = 1$  if the agent is an early consumer.<sup>3</sup> The

<sup>3</sup>Preferences of this form, which are similar to the one used by Diamond and Dybvig (1983), also appear in Bhattacharya and Gale (1987), Freeman (1988), Loewy (1991, 1995), and Qi (1994). In contrast, Jacklin (1987), Jacklin and Bhattacharya (1988), Chari and Jagannathan (1988), and Alonso (1996) assume that the preferences of both early and late consumers are defined over consumption in both the second and third period of life.

function  $u(\cdot)$  is assumed to be increasing, strictly concave, twice continuously differentiable, and to satisfy  $-cu''(c)/u'(c) > 1$  everywhere.

At  $t = 1$  there also exist the agents of generations  $-1$  and  $0$ . The former consists of measure  $1 - \lambda_1$  late consumers while the latter consists of measure  $\lambda_1$  early consumers and measure  $1 - \lambda_1$  late consumers. Since each member of these two generations knows his type at the beginning of time 1, each simply seeks to maximize his consumption,  $l(-1)$ ,  $e(0)$ , or  $l(0)$ .

The economy's illiquid technology, capital, may be accessed either directly or indirectly through an intermediary. Capital earns a fixed gross rate of return,  $x^L$ ,  $0 < x^L < 1$ , when held for one period and a random gross rate of return,  $\tilde{X}(t)$ , when held for two periods. For all  $t \geq 1$ ,  $\tilde{X}(t)$  is assumed to be an independent and identically distributed random variable whose realization is observed at the beginning of time  $t + 2$ . The prior distribution of  $\tilde{X}(t)$  satisfies  $\text{prob}\{\tilde{X}(t) = X^H\} = \phi$ ,  $\text{prob}\{\tilde{X}(t) = X^L\} = 1 - \phi$ , and  $E[\tilde{X}(t)] > 1$ , where  $\phi \in (0, 1)$ ,  $X^H = x^L x^H$ ,  $X^L = x^L x^L$ , and  $x^H$  necessarily satisfies  $x^H > 1/x^L$  and is given. Thus, ex ante, the technology shock raises the forward rate on time  $t$  capital per capita,  $k(t)$ , to  $x^H$  with probability  $\phi$  and leaves it unchanged at  $x^L$  with probability  $1 - \phi$ .

As in Jacklin and Bhattacharya (1988), Chari and Jagannathan (1988) and Alonso (1996), I assume that certain agents observe a signal  $s$  which contains information about the expected rate of return on capital held for two periods. Specifically, a proportion  $\alpha$  ( $0 < \alpha < 1$ ) of the late consumers of generation  $T$  ( $T$  large) begin time  $T + 1$  by observing  $s$  which determines (for them) a posterior distribution for  $\tilde{X}(T)$ .<sup>4</sup> Since  $X^H$  and  $X^L$  are given, let the posterior distribution on  $\tilde{X}(T)$  satisfy  $\text{prob}\{\tilde{X}(T) = X^H\} = \hat{\phi}_s$  and  $\text{prob}\{\tilde{X}(T) = X^L\} = 1 - \hat{\phi}_s$  when  $s$  is observed.

The parameter  $\hat{\phi}_s$  is assumed to be a random variable satisfying  $\text{prob}\{\hat{\phi}_s = \hat{\phi}_1\} = 1 - \pi$  and  $\text{prob}\{\hat{\phi}_s = \hat{\phi}_2\} = \pi$ .  $\hat{\phi}_1$ ,  $\hat{\phi}_2$ , and  $\pi$  are in turn assumed to imply that  $E(\hat{\phi}_s) = \phi$  and that if the posterior is characterized by  $\hat{\phi}_s = \hat{\phi}_2$ , then the informed late consumers of generation  $T$  prefer to withdraw prematurely rather than wait one more period to withdraw. In other words,  $\pi$  represents the probability that a bank run occurs at time  $T + 1$ .

Let  $\tilde{\lambda}$  be a random variable which corresponds to the measure of agents of generation  $T$  who withdraw at time  $T + 1$ . The above assumption on  $\hat{\phi}_1$ ,

<sup>4</sup>The issue of which generation(s) should include informed late consumers does not arise in the models of Jacklin and Bhattacharya (1988), Chari and Jagannathan (1988), and Alonso (1996) since each assumes the existence of a single generation. While it is possible for there to exist informed late consumers in each generation  $t \geq 1$ , I limit their existence to a single generation since the presence of fiat currency adds a degree of complexity to the economy's equilibrium not found in models without currency. Hence, the restriction serves to keep the analysis tractable.

$\hat{\phi}_2$ , and  $\pi$  implies that  $\text{prob}\{\tilde{\lambda} = \lambda_1\} = 1 - \pi$  and  $\text{prob}\{\tilde{\lambda} = \lambda_2\} = \pi$  where  $\lambda_1$  is the measure of agents who withdraw when no run occurs (early consumers only) and  $\lambda_2 \equiv \lambda_1 + \alpha(1 - \lambda_1)$  is the measure of agents who withdraw when a run does occur (early consumers plus informed late consumers).

In addition to capital, there exists a second asset, namely fiat currency. At  $t = 1$ , the economy's initial stock of  $M > 0$  units of currency is allocated across the members of generations  $-1$  and  $0$  as follows. Let  $\psi \in (0, 1)$  be given. Then the members of generation  $-1$  hold  $(1 - \psi)M/(2 - \psi)$  units of currency, the early consumers of generation  $0$  hold  $\psi M/(2 - \psi)$  units, and the late consumers of generation  $0$  hold the remaining  $(1 - \psi)M/(2 - \psi)$  units.<sup>5</sup> While the first two sets of agents inelastically supply their currency in exchange for time 1 good, the third set of agents does so in exchange for time 2 good. Since Friedman and Schwartz (1963) report that the stock of base money grew slowly during the 1929–1933 period, I assume that the stock of fiat currency is constant over time.

Finally, let  $p(t)$  be the time  $t$  value of currency in terms of time  $t$  good and  $m(t)$  be the time  $t$  per capita stock of currency. For nonnegative values of  $p(t)$  and  $m(t)$ , let  $q(t) = p(t)m(t)$  define per capita real balances. If  $p(t) > 0$  for all  $t \geq 1$ , then the one-period gross rate of return on time  $t$  real balances  $R(t) = p(t + 1)/p(t)$ .

### *Sequential Service*

Following Wallace (1988, 1990) and Chari (1989), I assume that the members of generations  $t \geq 1$  begin their second period of life physically isolated from one another.<sup>6</sup> After each agent has arrived at his specific location, he learns his type and then randomly and uniformly proceeds to a common central location which houses both the economy's bank and the market for fiat currency. Immediately upon his arrival at this central location, the agent either contacts the bank should he wish to withdraw or simply remains at the central location until next period should he wish to withdraw at that time. Note that the isolation assumption forces the bank to service the agents who choose to withdraw in the order that they arrive. This implies that a version of Diamond and Dybvig's (1983) sequential service constraint applies.

Middle-aged agents who choose to make time  $t$  withdrawals receive a

<sup>5</sup>If  $\psi = \lambda_1$ , then the stock of currency is allocated on an equal per capita basis across generations  $-1$  and  $0$ . However, I show below that it is necessary to assign a specific value to this parameter in order for there to exist a monetary equilibrium for this economy. Furthermore, this value exceeds  $\lambda_1$ .

<sup>6</sup>See Wallace (1988, 1990) for a defense of the isolation assumption.

payment of currency and/or capital that is consistent with the bank's optimal contract. Those whose withdrawals contain fiat currency proceed to the spot market where they exchange their currency for the time  $t$  good (see below). Once the spot market clears, these agents consume as do any others of their generation who may have received only capital. Both sets of agents then remain in the central location until the end of the next period.

The above scenario is slightly altered for generation  $T$ . Upon learning their type, the fraction  $\alpha$  of those agents who are late consumers immediately observe  $\phi_s$ . If these agents choose to withdraw, then they behave identically with the early consumers of their generation. If not, they behave identically with the uninformed late consumers of their generation. In the former case, sequential service implies that the bank cannot identify the existence of a run until it pays the marginal agent beyond measure  $\lambda_1$ . Thus, the optimal bank contract cannot provide these agents with the optimal risk sharing allocation (Diamond and Dybvig 1983, Prop. 1).

Simultaneous with the time  $t$  arrival of the members of generation  $t - 1$  at their isolated locations, the members of generation  $t - 2$  (all of whom are at the central location) and the bank observe the realization  $\tilde{X}(t - 2)$ . The bank makes payments of currency and/or capital consistent with its optimal contract to those agents of generation  $t - 2$  who wish to make time  $t$  withdrawals. As above, those agents receiving payments of currency proceed to the spot market. They, and any others who may have received only capital, consume once the spot market clears.<sup>7</sup>

Once all time  $t$  withdrawals have been completed, but prior to that period's spot market becoming active, the newly born members of generation  $t$  arrive at the central location. The bank then offers its optimal contract to these agents who in turn deposit their entire endowment, there being neither an incentive nor a mechanism to circumvent using the bank.<sup>8</sup> Assuming, as will be the case in equilibrium, that currency is part of the bank's optimal contract at every date, it follows that the demand for currency is well-defined. Consequently, the bank enters the time  $t$  spot market for currency on the part of its time  $t$  depositors. There it is joined by those agents

<sup>7</sup>Recall that the agents of generations  $-1$  and  $0$  are assumed to know their type at time 1. Thus, it is assumed further that both sets of agents are at the central location at the beginning of time 1. Given that they are endowed with currency, these agents' only activity is to enter either the time 1 or time 2 spot market for currency depending upon the period in which they prefer to consume.

<sup>8</sup>The insurance aspect of the bank's optimal contract explains the lack of incentive. As for the lack of a mechanism, the type of destabilizing trade in deposits described by Jacklin (1987, 1989) cannot arise here because they necessarily entail trading claims backed by capital. These in turn require having to verify an agent's type.

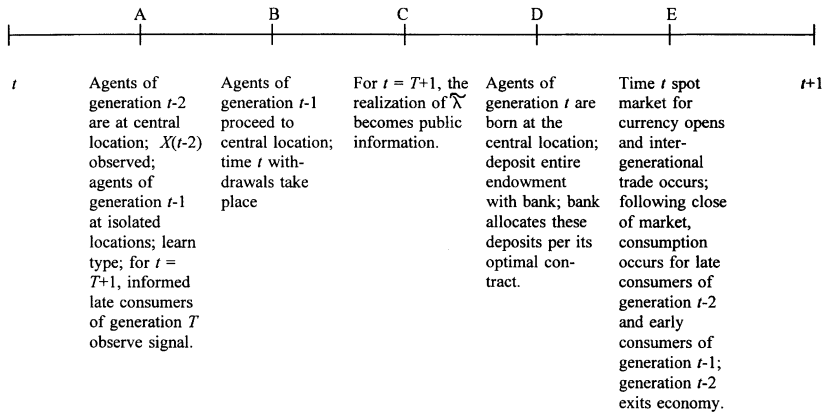


Figure 1.

of generations  $t-2$  and  $t-1$  who wish to trade currency for time  $t$  good. Following trade in the spot market and the subsequent consumption of the generation  $t-2$  late consumers and the generation  $t-1$  early consumers, period  $t$  concludes. Figure 1 provides a summary of the order in which the events comprising time  $t$  occur.

Note that the spot market for currency effectively operates as in any model of overlapping generations, namely it provides for intergenerational transfers. Within the context of this model, it also serves as a mechanism for intergenerational banking similar to those of Bryant (1981) and Qi (1994). This similarity notwithstanding, note that Bryant does not model banks as providers of liquidity while the transfers in Qi are internal to the bank and arise in an economy in which only real assets appear.

### 3. The Bank's Problem

The sequence of events comprising period  $t$  implies that the only assets that the bank has available to finance withdrawals by the members of generations  $t - 2$  and  $t - 1$  are the currency and capital that it holds on behalf of these two generations. While this structure permits, for example, using the deposits held on behalf of generation  $t - 2$  to help finance withdrawals by members of generation  $t - 1$ , I assume instead that the bank arranges its portfolio on a generation-by-generation basis. Therefore, the bank finances the time  $t$  withdrawals of generations  $t - 2$  and  $t - 1$  using only

the assets that were originally deposited by the members of those generations.<sup>9</sup> This restriction does not preclude the presence of intergenerational transfers since they continue to arise through the spot market for currency.

Why assume that the bank's portfolio is generation specific?<sup>9</sup> The primary reason for doing so is that it serves to keep the model tractable given the implications of including fiat currency. For example, the presence of two distinct assets in the bank's portfolio requires that each be accounted for in the financing of withdrawals. Since these decisions are endogenous to the bank, this aspect of the bank's problem is significantly simplified under generation-specific banking. This simplification in turn implies that the analysis of the interaction between the optimal bank contract and the market for real balances, though complex, remains manageable.

In order to treat real balances and capital as distinct assets, the only withdrawals that are admissible are those that can be financed using anywhere from 0 to 100% of each of the two assets that the bank has on hand at the time that withdrawals are made.<sup>10</sup> This in turn implies that at time  $t$  the bank must choose the share of each asset that it will use to finance the withdrawals of generation  $t$ . Furthermore, the structure of the bank's portfolio requires that the shares of each asset sum to one across time  $t + 1$  and  $t + 2$  withdrawals.

Let  $\chi(t)$  and  $\kappa(t)$  be the shares of currency and capital that the bank offers to the measure  $\lambda_1$  middle-aged agents of generation  $t \neq T$  who arrive seeking to withdraw. For generation  $T$ , let these shares be defined by  $\chi_1(T)$  and  $\kappa_1(T)$ . Should a generation  $T$  middle-aged agent beyond measure  $\lambda_1$  arrive and seek to withdraw (thereby signaling a bank run), let  $\chi_2(T)$  and  $\kappa_2(T)$  be the shares of currency and capital offered to the next measure  $\lambda_2 - \lambda_1 = \alpha(1 - \lambda_1)$  middle-aged agents who withdraw.<sup>11</sup> The bank is assumed to suspend payments once measure  $\lambda_2$  middle-aged agents have withdrawn.

Since the bank can finance the withdrawals of the late arriving middle-aged agents of generation  $T$  differently from those who arrived earlier, it follows that the resulting quantity of consumption across these two groups of agents will typically differ. Furthermore, since a run increases the rate of

<sup>9</sup>Freeman (1988) offers an effectively equivalent specification. He assumes that a bank is generation specific, it being formed by the young agents of each generation. Thus, Freeman has a sequence of finite-lived banks while here the bank is infinite-lived, but structures its portfolio on a generation-specific basis.

<sup>10</sup>This feature rules out the bank offering withdrawals that require it to issue more currency or capital than it has on hand by "transforming" one asset into the other.

<sup>11</sup>Instead of treating all additional  $\lambda_2 - \lambda_1$  agents the same, the bank could continuously vary its payments across agents. However, because agents are identical ex ante, it is optimal for the bank to treat them identically.



return on real balances, even the consumption of the first group to arrive will differ depending upon whether a run occurs or not. To this end, let  $e_1(T)$  denote the quantity of consumption when no run occurs. Likewise, let  $e_2^h(T)$ ,  $h = 1, 2$ , denote the quantity of consumption when a run occurs and the agent is among the first group (of measure  $\lambda_1$ ) or the second group (of measure  $\lambda_2 - \lambda_1$ ) of agents to withdraw.

Last, recall that the consumption of a late consumer of generation  $t \geq 1$  depends upon the realization of  $\tilde{X}(t)$ . Therefore, let  $l^j(t)$ ,  $j = H, L$ , denote the time  $t + 2$  consumption of such an agent when  $\tilde{X}(t) = X^H$  or  $X^L$ . Should this value be contingent on the realization of  $\tilde{\lambda}$ , the notation  $l_i^j(t)$ ,  $i = 1, 2$ ,  $j = H, L$ , will be used.

Being a zero profit, zero cost entity, the bank's objective is to maximize the ex ante expected utility of private agents. In the present case, this entails taking expectations with respect to agent type, the technology shock, and the possibility of a bank run occurring at time  $T + 1$ . For the latter, however, since the realization of  $\tilde{\lambda}$  is observed prior to the bank offering its contract to the members of generation  $T + 1$  (see Figure 1), it follows that for generations born at time  $T + 1$  and later, the bank conditions upon the realization of  $\tilde{\lambda}$  rather than by taking expectations.

Given the admissibility constraints discussed above, the bank solves the following set of problems:

Problem  $B(t)$ ,  $t \neq T - 1, T$ :

Choose  $\{e(t), l^j(t) [j = H, L], \chi(t), \kappa(t), q(t), k(t)\}$  to maximize

$$U_B(t) = \lambda_1 u[e(t)] + (1 - \lambda_1) \{ \phi u[l^H(t)] + (1 - \phi) u[l^L(t)] \}, \quad (1a)$$

subject to

$$\lambda_1 e(t) = \chi(t) R(t) q(t) + \kappa(t) x^L k(t), \quad (1b)$$

$$(1 - \lambda_1) l^j(t) = [1 - \chi(t)] R(t + 1) R(t) q(t) + [1 - \kappa(t)] X^j k(t); \\ j = H, L, \quad (1c)$$

$$q(t) + k(t) = 1; \quad (1d)$$

$$0 \leq \chi(t), \kappa(t) \leq 1; \quad (1e)$$

$$\phi u[l^H(t)] + (1 - \phi) u[l^L(t)] \geq u[R(t + 1) e(t)]; \quad (1f)$$

taking  $R(t)$  and  $R(t + 1)$  as given.

Problem  $B(T - 1)$ :

Choose  $\{e(T - 1), l_i^j(T - 1)[i = 1, 2; j = H, L], \chi(T - 1), \kappa(T - 1), q(T - 1), k(T - 1)\}$  to maximize

$$\begin{aligned} U_B(T - 1) = & \lambda_1 u[e(T - 1)] + (1 - \lambda_1)\{(1 - \pi)[\phi u[l_1^H(T - 1)]] \\ & + (1 - \phi)u[l_1^L(T - 1)]\} + \pi[\phi u[l_2^H(T - 1)]] \\ & + (1 - \phi)u[l_2^L(T - 1)]\}, \end{aligned} \quad (2a)$$

subject to

$$\lambda_1 e(T - 1) = \chi(T - 1)R(T - 1)q(T - 1) + \kappa(T - 1)x^L k(T - 1); \quad (2b)$$

$$\begin{aligned} (1 - \lambda_1)l_i^j(T - 1) = & [1 - \chi(T - 1)]R_i(T)R(T - 1)q(T - 1) \\ & + [1 - \kappa(T - 1)]X^j k(T - 1); \\ & i = 1, 2; \quad j = H, L; \end{aligned} \quad (2c)$$

$$q(T - 1) + k(T - 1) = 1; \quad (2d)$$

$$0 \leq \chi(T - 1), \kappa(T - 1) \leq 1; \quad (2e)$$

$$\begin{aligned} (1 - \pi)\{\phi u[l_1^H(T - 1)] + (1 - \phi)u[l_1^L(T - 1)]\} + \pi\{\phi u[l_2^H(T - 1)] \\ + (1 - \phi)u[l_2^L(T - 1)]\} \geq (1 - \pi)u[R_1(T)e(T - 1)] \\ + \pi u[R_2(T)e(T - 1)]; \end{aligned} \quad (2f)$$

taking  $R(T - 1)$ ,  $R_1(T)$ , and  $R_2(T)$  as given with  $R_1(T)$ ,  $R_2(T)$  corresponding to  $\tilde{\lambda} = \lambda_1, \lambda_2$ .

Problem  $B(T)$ :

Choose  $\{e_1(T), e_2^h(T), l_i^j(T), \chi_i(t), \kappa_i(t)[h, i = 1, 2; j = H, L], q(T), k(T)\}$  to maximize

$$\begin{aligned} U_B(T) = & (1 - \pi)(\lambda_1 u[e_1(T)] + (1 - \lambda_1)\{\phi u[l_1^H(T)] + (1 - \phi)u[l_1^L(T)]\}) \\ & + \pi(\lambda_1 u[e_2^1(T)] + (\lambda_2 - \lambda_1)u[e_2^2(T)] + (1 - \lambda_2)\{\phi u[l_2^H(T)] \\ & + (1 - \phi)u[l_2^L(T)]\}), \end{aligned} \quad (3a)$$

subject to

$$\lambda_1 e_1(T) = \chi_1(T) R_1(T) q(T) + \kappa_1(T) x^L k(T); \quad (3b)$$

$$(1 - \lambda_1) l_1^j(T) = [1 - \chi_1(T)] R_1(T+1) R_1(T) q(T) + [1 - \kappa_1(T)] X^j k(T);$$

$$j = H, L; \quad (3c)$$

$$\lambda_1 e_2^1(T) = \chi_1(T) R_2(T) q(T) + \kappa_1(T) x^L k(T); \quad (3d)$$

$$(\lambda_2 - \lambda_1) e_2^2(T) = \chi_2(T) R_2(T) q(T) + \kappa_2(T) x^L k(T); \quad (3e)$$

$$(1 - \lambda_2) l_2^j(T) = [1 - \chi_1(T) - \chi_2(T)] R_2(T+1) R_2(T) q(T)$$

$$+ [1 - \kappa_1(T) - \kappa_2(T)] X^j k(T); j = H, L; \quad (3f)$$

$$q(T) + k(T) = 1; \quad (3g)$$

$$0 \leq \chi_i(t), \kappa_i(t) \leq 1, i = 1, 2; 0 \leq \chi_1(T) + \chi_2(T) \leq 1; 0 \leq \kappa_1(T)$$

$$+ \kappa_2(T) \leq 1; \quad (3h)$$

$$(1 - \pi)\{\phi u[l_1^H(T)] + (1 - \phi)u[l_1^L(T)]\} + \pi\{\phi u[l_2^H(T)] + (1 - \phi)u[l_2^L(T)]\}$$

$$\geq (1 - \pi)u[R_1(T+1)e_1(T)] + \pi\{\Lambda u[R_2(T+1)e_2^1(T)]$$

$$+ (1 - \Lambda)u[R_2(T+1)e_2^2(T)]\}; \quad (3i)$$

taking  $R_i(T)$  and  $R_i(T+1)$ ,  $i = 1, 2$ , as given where  $R_1(T+1)$  and  $R_2(T+1)$  represent the cases  $\tilde{\lambda} = \lambda_1, \lambda_2$ , and  $\Lambda = \lambda_1/\lambda_2$  is the fraction of agents withdrawing early who are early consumers.

Note that for  $t \leq T - 2$ , the expectation with respect to  $\tilde{\lambda}$  enters Problem  $B(t)$  implicitly through the values of  $R(t)$  and  $R(t+1)$ . In Problems  $B(T-1)$  and  $B(T)$ , the expectation with respect to  $\tilde{\lambda}$  enters both through rates of return and by altering the set of choice variables to allow for state contingent consumptions. In addition, it implies that the share parameters in Problem  $B(T)$  are also state contingent. Given their importance in what follows, these contingent relationships explicitly appear in the notation. For  $t \geq T+1$ , each of Problem  $B(t)$ 's choice variables is contingent on the realization of  $\tilde{\lambda}$ . By market clearing, this in turn implies that for  $t \geq T$ ,  $R(t)$  is also contingent upon this realization. To avoid notational clutter, this dependence is subsumed for  $t \geq T+1$  except for  $R_i(T+1)$  where its presence is needed to clarify the both the definitions of  $l_i^j(T)$  above and the time  $T+2$  market clearing conditions given below.

Equations (1b)–(1c), (2b)–(2c), and (3b)–(3c) illustrate how the bank allocates its assets to finance the withdrawals of generation  $t \geq 1$  when  $\lambda_1$

middle-aged agents seek to withdraw. In particular, the bank can only finance time  $t + 2$  withdrawals using the quantities of currency and capital that are still available once all time  $t + 1$  withdrawals have been completed. Therefore, at time  $t + 2$  the bank offers late consumers the remaining shares of currency,  $1 - \chi(t)$ , and capital,  $1 - \kappa(t)$ .<sup>12</sup> Equations (3d)–(3f) show how the bank allocates its assets to finance the withdrawals of generation  $T$  when a run occurs. Since it offers the shares  $\chi_2(T)$  and  $\kappa_2(T)$  to those middle-aged agents who withdraw after the first measure  $\lambda_1$  have done so, it follows that at time  $T + 2$  the bank offers the remaining shares,  $1 - \chi_1(T) - \chi_2(T)$  and  $1 - \kappa_1(T) - \kappa_2(T)$ , to that generation's uninformed late consumers. Finally, Equations (1d)–(1e), (2d)–(2e), and (3g)–(3h) describe how the bank allocates each agent's endowment and the necessary corner conditions on the share parameters.

Conditional on having learned their type at the beginning of time  $t + 1$ , the only members of generation  $t$  who may have an incentive to withdraw in a period other than the one in which they consume are those agents who learn that they are late consumers. In order to rule out such behavior, the time  $t$  bank contract must include a constraint which induces late consumers to truthfully reveal their type. This constraint, which is given by either Equation (1f), (2f), or (3i) depending upon the value of  $t$ , imply that given the prior distribution for  $\tilde{X}(t)$ , each late consumer of generation  $t$  must weakly prefer to withdraw in period  $t + 2$  rather than do so in period  $t + 1$  and then save the proceeds of his withdrawal in order to finance his consumption during the next period.<sup>13</sup> Of course, by definition the self-selection constraint for generation  $T$  must also hold when informed late consumers observe  $\hat{\phi}_s = \hat{\phi}_1$  and fail to do so when they observe  $\hat{\phi}_s = \hat{\phi}_2$ .

#### 4. Equilibrium

Given the economy's initial conditions, market clearing at  $t = 1$  and  $t = 2$  satisfy

$$q(1) = p(1) \left[ \frac{\psi}{2 - \psi} + \frac{1 - \psi}{2 - \psi} \right] M; \tag{4}$$

$$q(2) = \chi(1)R(1)q(1) + p(2)(1 - \psi)M/(2 - \psi). \tag{5}$$

For  $3 \leq t \leq T$  and  $T + 3 \leq t$ , we have that

<sup>12</sup>The subscript which appears for generation  $T$  has been dropped here to simplify matters.

<sup>13</sup>Note that Equations (1f), (2f), and (3i) assume that if a late agent were to withdraw prematurely, then it is optimal for him to save his entire withdrawal in the form of real balances. As shown below, this behavior derives from real balances dominating capital in rate of return when each is held for one period.

$$q(t) = \chi(t-1)R(t-1)q(t-1) + [1 - \chi(t-2)]R(t-1)R(t-2)q(t-2). \quad (6)$$

Since a run may or may not occur at time  $T+1$ , the market for real balances at that date satisfies one of the following two expressions:

$$q(T+1) = \chi_1(T)R_1(T)q(T) + [1 - \chi(T-1)]R_1(T)R(T-1)q(T-1); \quad (7a)$$

$$\begin{aligned} q(T+1) &+ [\kappa_1(T) + \kappa_2(T)]x^L k(T)(1 - \Lambda) \\ &= [\chi_1(T) + \chi_2(T)]R_2(T)q(T)\Lambda \\ &+ [1 - \chi(T-1)]R_2(T)R(T-1)q(T-1); \end{aligned} \quad (7b)$$

where recall that  $\Lambda = \lambda_1/\lambda_2$  is the fraction of agents withdrawing early who are early consumers. Finally, the occurrence or not of a run at time  $T+1$  implies that market clearing at time  $T+2$  must satisfy one of the following two expressions:

$$\begin{aligned} q(T+2) &= \chi(T+1)R_1(T+1)q(T+1) \\ &+ [1 - \chi_1(T)]R_1(T+1)R_1(T)q(T); \end{aligned} \quad (8a)$$

$$\begin{aligned} q(T+2) &= \chi(T+1)R_2(T+1)q(T+1) \\ &+ R_2(T+1)[\kappa_1(T) + \kappa_2(T)]x^L k(T)(1 - \Lambda) \\ &+ [\chi_1(T) + \chi_2(T)]R_2(T+1)R_2(T)q(T)(1 - \Lambda) \\ &+ [1 - \chi_2(T) - \chi_2(T)]R_2(T+1)R_2(T)q(T). \end{aligned} \quad (8b)$$

Equations (7a) and (8a) correspond to the case in which no run occurs at time  $T+1$ . Hence, these expressions are qualitatively the same as Equation (6) in as much as the demand for real balances is solely due to the young agents of generation  $t$  while the supply is due to the early consumers of generation  $t-1$  and the late consumers of generation  $t-2$ .

Equation (7b) describes time  $T+1$  market clearing in the event that a run occurs during that period. The second term on the left-hand side represents the demand for real balances on the part of the informed late consumers of generation  $T$  while the supply derives from the early consumers of generation  $T-1$  and the late consumers of generation  $T-2$ . Demand increases because informed late consumers choose to exchange the part of their withdrawal in the form of capital output for currency. This

exchange occurs for two reasons. First, since the bank can identify an agent's age, it is not possible to redeposit currency or capital output into the bank. Hence, only the agent can save the proceeds of his withdrawal. As shown in the next section, the equilibrium rate of return on real balances dominates that of capital when each is held for one period so that all such saving is in the form of real balances. Second, since it is not possible to identify an agent's type, neither currency nor goods can be used to purchase a claim backed by capital.

Finally, Equation (8b) depicts the market clearing condition at time  $T + 2$  when a run has occurred in the previous period. The left-hand side and the first term on the right-hand side are as in Equation (6). As for the remaining terms on the right-hand side, the second and third correspond to the supply of real balances on the part of the informed late consumers of generation  $T$ . The fourth term represents the supply of real balances on the part of the uninformed late consumers of generation  $T$ . More specifically, the second term represents the part of the withdrawal of informed late consumers which was originally in the form of capital output and was then exchanged for currency. Likewise, the third term represents the part of the withdrawal which was originally currency and remained as such.

I am now in a position to define an equilibrium.

**DEFINITION 1.** *Given  $M > 0$ , the random variable  $\tilde{\lambda}$ , and the stochastic process  $\{\tilde{X}(t)\}_{t=1}^{\infty}$ , an equilibrium consists of stochastic processes  $\{p(t), q(t), k(t)\}_{t=1}^{\infty}$ ,  $\{e(1), l^H(1), l^L(1), \dots, e(T-1), l_1^H(T-1), l_1^L(T-1), l_2^H(T-1), l_2^L(T-1), e_1(T), e_2(T), e_2^2(T), l_1^H(T), l_1^L(T), l_2^H(T), l_2^L(T), e(T+1), l^H(T+1), l^L(T+1), \dots\}$ , and  $\{\chi(1), \kappa(1), \dots, \chi_1(T), \kappa_1(T), \chi_2(T), \kappa_2(T), \chi(T+1), \kappa(T+1), \dots\}$  such that for every  $t \geq 1$ ,  $p(t) > 0$ , Problem B(t) is satisfied, and the spot market for currency clears.*

### 5. Simulation and the "Stylized Facts"

Given the degree of complexity of the model, the most direct way of showing that the equilibrium just defined implies behavior that is consistent with the "stylized facts" is to simulate an equilibrium. Even so, since the possibility that a run occurs at time  $T + 1$  affects the bank's problem at every date, devising an equilibrium that can be solved numerically requires some simplification. Specifically, since  $T$  is taken to be large, I assume that the equilibrium for periods far removed from time  $T + 1$  can be well-approximated by a stationary equilibrium in which a bank run never occurs.

Consider then a version of the economy in which a run never occurs. In such an economy, the bank maximizes Equation (1a) subject to Equations (1b)–(1f) for each date  $t \geq 1$ . Market clearing is then described by Equations

(4)–(6) where the latter now holds for all  $t \geq 3$ . Since the stock of fiat currency is fixed, there typically will exist a stationary equilibrium in which  $R(t) = 1$  for all  $t \geq 1$ . Assuming this to be the case, since  $x^L < 1$  and  $X^L < 1 < X^H$ , rate of return dominance and risk sharing imply that the solution to this problem satisfies  $0 < q^* < 1$ ,  $0 < \chi^* < 1$ , and  $\kappa^* = 0$  for all  $t$ . Finally, in order that the market for real balances clears at  $t = 1$  and 2, it is necessary to impose that  $\psi = \chi^*$ .

The simulated equilibrium was constructed as follows. First, the stationary equilibrium is computed. Next, since the software (*MathCAD 5.0*) can solve a maximum of 50 equations in 50 unknowns, the stationary equilibrium is assumed to apply to all variables dated  $1 \leq t \leq T - 3$  and  $T + 3 \leq t$ . Thus, the program solves Problems  $B(T - 2)$  through  $B(T + 1)$  plus solves for  $R(T - 2)$  through  $R(T + 2)$ .<sup>14</sup> However, while the approximation guarantees that the market for real balances clears for  $1 \leq t \leq T - 3$  and  $T + 5 \leq t$ , generational overlap and consistency with the approximation imply that the simulation can only guarantee that the market clears for  $T - 1 \leq t \leq T + 3$ . To see why, consider the market clearing conditions for  $T - 2, T - 1, T + 3$ , and  $T + 4$  when consistency with the approximation is assumed:

$$q(T - 2) = \chi^* \cdot 1 \cdot q^* + (1 - \chi^*) \cdot 1 \cdot 1 \cdot q^* ; \quad (9)$$

$$q(T - 1) = \chi(T - 2)R(T - 2)q(T - 2) + (1 - \chi^*)R(T - 2) \cdot 1 \cdot q^* ; \quad (10)$$

$$q^* = \chi(T + 2)R(T + 2)q(T + 2) + [1 - \chi(T + 1)]R(T + 2)R(T + 1)q(T + 1) ; \quad (11)$$

$$q^* = \chi^* \cdot 1 \cdot q^* + [1 - \chi(T + 2)] \cdot 1 \cdot R(T + 2)q(T + 2) .^{15} \quad (12)$$

It is clear that if Equations (10) and (11) are assumed to be satisfied in the simulation (which they are), then Equations (9) and (12) typically will not be. Consequently, the extent to which each of these expressions fails to hold can be viewed as a gauge of the accuracy of the simulation relative to the “actual” equilibrium.

In the simulation described below, I assume that  $u(c) = c^{1-\gamma}/(1 - \gamma)$ . Next, recall that it is not the parameters of the posterior distribution on  $\tilde{X}(T)$  per se that play a role in the bank’s problem, but instead is simply the probability that either  $\hat{\phi}_1$  or  $\hat{\phi}_2$  occurs. Therefore, the economy’s parameter

<sup>14</sup>A copy of the program is available from the author upon request.

<sup>15</sup>The market clearing conditions for  $T \leq t \leq T + 2$  are as in Equations (6), (7), and (8).

vector is given by  $(\pi, \phi, \lambda_1, \lambda_2, x^H, x^L, \gamma)$ . Table 1 presents results for an economy in which  $(\pi, \phi, \lambda_1, \lambda_2, x^H, x^L, \gamma) = (0.1, 0.9, 0.5, 0.75, 4.389, 0.5, 2)$ . These parameters in turn imply that  $E(\tilde{X}) = 2$  and  $\alpha = 0.5$ . Therefore, a bank run does *not* occur at time  $T + 1$  with probability 0.9 in which case 50% of the middle-aged consumers (all of whom are early consumers) withdraw. With probability 0.1, an additional 50% of the remaining middle-aged consumers (all of whom are late consumers) withdraw as well. By assumption, the bank suspends withdrawals once 75% of the middle-aged agents have withdrawn.

Table 1 details the results of the simulation. Panel A of the table provides equilibrium values for consumption, real balances, and the share parameters under the stationary approximation. Hence, these values apply for  $1 \leq t \leq T - 3$  and  $t \geq T + 3$ . Panels B–H of the table provide similar results for dates  $T - 2$  through  $T + 2$ . Panel I shows the stationary return on real balances,  $R^*$ , followed by the simulated returns for  $T - 2 \leq t \leq T + 2$ .<sup>16</sup> Panel J of the table verifies that under the prior distribution for  $\tilde{X}(t)$  the self-selection constraint does not bind at any date. For each date listed in the panel’s top row, the bottom row measures the difference between the left- and right-hand sides of the associated self-selection constraint. Recall that a bank run occurs when the time  $T$  self-selection constraint, Equation (3i), fails. Direct calculation shows that any  $(\hat{\phi}_1, \hat{\phi}_2)$  such that  $\hat{\phi}_1 \in (0.914, 1)$  and  $\hat{\phi}_2 = 8.977(1 - \hat{\phi}_1)$  is consistent with  $\pi = 0.1$ ,  $E(\hat{\phi}_s) = \phi = 0.9$ , and the failure of Equation (3i) to hold should  $\hat{\phi}_s = \hat{\phi}_2$ .

How accurate of a description of the economy’s equilibrium are the values reported in Table 1? By Equation (9),  $q(T - 2)$  should equal  $q^* = 0.685$ . Instead, as shown in Panel B,  $q(T - 2) = 0.689$ , an error of 0.6%. On the other hand, the right-hand side of Equation (12) implies that  $q(T + 4)$  equals 0.690 when  $\tilde{\lambda} = \lambda_1$  and equals 0.707 when  $\tilde{\lambda} = \lambda_2$ . Since the approximation requires that  $q(T + 4) = q^* = 0.685$ , these represent errors of 0.7% and 3.2%, respectively. Given the size of these errors, the simulation appears to provide quite an accurate description of the economy’s “actual” equilibrium.<sup>17</sup>

Does the simulated equilibrium reported in Table 1 deliver the features needed to successfully confront the “stylized facts” of the 1929–1933 period? Examination of Panels D and I show that this question can be answered in the affirmative. Specifically, the time  $T$  optimal bank contract calls

<sup>16</sup>As with the subscripts on  $R_i(T + 1)$  introduced above, the terms  $R_1(T + 2)$  and  $R_2(T + 2)$  in Table 1 correspond to the cases  $\tilde{\lambda} = \lambda_1, \lambda_2$ .

<sup>17</sup>The fact that the error at time  $T + 4$ ,  $\tilde{\lambda} = \lambda_2$ , is about five times greater than the other two errors is a reflection of the fact that the simulation is forcing the length of time that the economy needs to adjust to the run to be less than the time it would need in the “actual” equilibrium.



for (i) a partial suspension of withdrawals should a run occur,  $e_2^1(T) = 1.199 > 0.816 = e_2^2(T)$ ; (ii) potentially productive loans are called to finance withdrawals should a run occur,  $\kappa_2(T) > 0$ ; and (iii) the price level declines,  $R_2(T) = 1.115$ .<sup>18</sup> If the magnitude of the run is increased, say  $\lambda_2 = 0.9$ , then the magnitude of the deflation increases as well with  $R_2(T) = 1.322$ .<sup>19</sup>

More generally, how do we account for the other features of this equilibrium? First, since the simulated values for  $R(t) \approx 1$  for  $t \neq T$ , the rate of return dominance and risk sharing arguments which explain the form of the optimal bank contract under the stationary approximation, also apply for  $T - 2$ ,  $T - 1$ ,  $T + 1$ , and  $T + 2$ . Hence, it is again the case that the bank's optimal portfolio is diversified,  $0 < q(t) < 1$ , while  $\kappa(t) = 0 < \chi(t) < 1$  for these particular periods.<sup>20</sup>

Second, consider the outcomes for  $t = T$ . Applying the rate of return dominance and risk sharing arguments to this case imply that  $0 < q(T)$ ,  $\chi_1(T)$ ,  $\chi_2(T) < 1$  and  $\kappa_1(T) = 0$ . Furthermore, we have that  $\chi_1(T) + \chi_2(T) = 1$ . This implies that were it possible to continue to finance the withdrawals of late arriving middle-aged agents with currency, the bank would have done so. However, because this is not possible, the bank begins to call loans instead. Hence,  $\kappa_2(T) > 0$ .

Third, Panel I of Table 1 shows that a deflation occurs between periods  $T - 1$  and  $T$ , one period *before* the period in which a run can occur. That  $R(T - 1)$  exceeds (by more than a trivial amount) one occurs for the following reason. Risk sharing and the fact that a run causes  $R_2(T)$  to exceed  $R_1(T)$  imply that  $q(T)$  exceeds the demand for real balances at all other dates. By Equation (6), this in turn implies that  $R(T - 1)$  also increases. Should a run in fact occur at time  $T$ , then the deflation worsens and is followed by a cyclical adjustment of the rate of return back to one. Should no run occur at time  $T$ , then the rate of return immediately falls and is again followed by a cyclical adjustment back to one. As suggested by the errors in matching the value of  $q^*$  at time  $T + 4$ , it is not surprising to find that the effects, both expected and realized, from the possibility of a run dissipate more quickly when the run does not occur than when it does.

<sup>18</sup>Since this value exceeds  $x^L$ , this verifies the claim made earlier that during a run, informed late consumers choose to convert all of their withdrawal in the form of capital output into currency.

<sup>19</sup>Copies of the complete results for this simulation as well as for the three others discussed below are available from the author upon request.

<sup>20</sup>Qi (1994) shows that the optimality of a diversified portfolio can depend upon the specification of initial conditions. In his model, the bank holds a diversified portfolio when the initial old generation is not endowed with real goods while the opposite occurs if it is endowed with goods held in the illiquid asset. Since the initial old and middle-aged in this paper are endowed with only fiat currency, a result comparable to the former obtains.

TABLE 1. *Simulation Results*


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$(\pi, \phi, \lambda_1, \lambda_2, x^H, x^L, \gamma) = (0.1, 0.9, 0.5, 0.75, 4.389, 0.5, 2)$

Panel A:  $1 \leq t \leq T - 3; t \geq T + 3$       Panel B:  $t = T - 2$

$e^*$	$l^{H*}$	$l^{L*}$	$q^*$	$\chi^*$	$\kappa^*$	$e$	$l^H$	$l^L$	$q$	$\chi$	$\kappa$
1.089	1.663	0.439	0.685	0.794	0	1.094	1.660	0.452	0.689	0.791	0

Panel C:  $t = T - 1$

$e$	$l_1^H$	$l_1^L$	$l_2^H$	$l_2^L$	$q$	$\chi$	$\kappa$
1.112	1.666	0.454	1.701	0.489	0.688	0.788	0

Panel D:  $t = T$

$e_1$	$l_1^H$	$l_1^L$	$e_2^1$	$e_2^2$	$l_2^H$	$l_2^L$	$q$	$\chi_1$	$\chi_2$	$\kappa_1$	$\kappa_2$
1.072	1.616	0.468	1.199	0.816	0.280	0.260	0.705	0.763	0.237	0	0.121

Panel E:  $t = T + 1\tilde{\lambda} = \lambda_1$

$e$	$l^H$	$l^L$	$q$	$\chi$	$\kappa$
1.053	1.649	0.424	0.685	0.801	0

Panel F:  $t = T + 1\tilde{\lambda} = \lambda_2$

$e$	$l^H$	$l^L$	$q$	$\chi$	$\kappa$
0.936	1.601	0.374	0.684	0.823	0

Panel G:  $t = T + 2\tilde{\lambda} = \lambda_1$

$e$	$l^H$	$l^L$	$q$	$\chi$	$\kappa$
1.104	1.668	0.449	0.686	0.791	0

Panel H:  $t = T + 2\tilde{\lambda} = \lambda_2$

$e$	$l^H$	$l^L$	$q$	$\chi$	$\kappa$
1.154	1.684	0.480	0.690	0.780	0

Panel I: Rates of Return on Real Balances

$R^*$	$R(T - 2)$	$R(T - 1)$	$R_1(T)$	$R_2(T)$	$R_1(T + 1)$	$R_2(T + 1)$	$R_2(T + 2)$	$R_2(T + 2)$
1	1.003	1.025	0.997	1.115	0.960	0.831	1.017	1.072

Panel J: Slackness in Self-Selection Constraint

$1 \leq t \leq T - 4$	$T - 3$	$T - 2$	$T - 1$	$T$	$T + 1\tilde{\lambda} = \lambda_1$	$T + 1\tilde{\lambda} = \lambda_1$	$T + 2\tilde{\lambda} = \lambda_1$	$T + 2\tilde{\lambda} = \lambda_1$	$T + 3 \leq t$
0.149	0.147	0.128	0.135	0.219	0.152	0.167	0.144	0.124	0.149

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Fourth, while the majority of features of the equilibrium are robust to changes in the parameters, two in particular,  $\chi_1(T) + \chi_2(T) = 1$  and  $\kappa_2(T) > 0$  are not. Indeed, changing a single parameter of the model is sufficient to cause any of the other three possible permutations to arise: (i)  $\chi_1(T) + \chi_2(T) = 1$ ,  $\kappa_2(T) = 0$ ; (ii)  $\chi_1(T) + \chi_2(T) < 1$ ,  $\kappa_2(T) = 0$ ; and (iii)  $\chi_1(T) + \chi_2(T) < 1$ ,  $\kappa_2(T) > 0$ . Cases (i) and (ii) arise by raising  $\pi$ , the probability that a run occurs, up to and then beyond the threshold value of 0.221. As  $\pi$  increases, the optimal quantity of real balances increases for  $T - 2 \leq t \leq T$ . This in turn implies that  $\chi(T - 2)$ ,  $\chi(T - 1)$ , and  $\chi_1(T)$  fall. The latter then permits  $\chi_2(T)$  to increase. However, because these changes cause a further increase in  $R(T - 1)$  (for the reason given above), time  $T + 1$  market clearing implies that  $R_2(T)$  does not increase as much as before. Since this loosens the extent to which the return on real balances dominates capital, the bank first reduces  $\kappa_2(T)$  to zero followed by it choosing to set  $\chi_1(T) + \chi_2(T)$  less than one.

Finally, case (iii) occurs following a sufficiently large increase in  $x^H$ . Since this increases the riskiness of capital, optimal risk sharing implies that both the demand for real balances and the share parameter for currency increase at each date. In particular, the bank increases  $\chi_1(T)$  and so must reduce  $\chi_2(T)$ . Given that  $l_2^H(T)$  is unusually large relative to  $l_2^L(T)$  due to the large value of  $x^H$ , optimal risk sharing also leads the bank to substitute called loans,  $\kappa_2(T)$ , for additional units of currency as a means to finance late withdrawals,  $e_2^2(T)$ . In this particular case, the decline in  $\chi_2(T)$  is large enough that  $\chi_1(T) + \chi_2(T) < 1$ .

## 6. Conclusion

This paper offers a version of Jacklin and Bhattacharya's (1988) model of information-based bank runs that can confront certain evidence on bank runs during the 1929–1933 period reported by Friedman and Schwartz (1963), Cagan (1965), Bernanke (1983), and others. In particular, I use a tractable approximation of the model's equilibrium to show that (i) a bank run causes concurrent deflation; (ii) the deflation increases with the magnitude of the run; (iii) banks partially suspend withdrawals once they recognize that a run is occurring; and (iv) runs may cause banks to call potentially productive loans to finance certain withdrawals. While these results are encouraging, they must necessarily be viewed as a first step towards the larger goal of analyzing the model's "actual" equilibrium. This project is left for future research.

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