

# *Equilibrium Policy with Dynamically Naive Agents\**

Within the context of a two-period-lived overlapping generations monetary economy, I define an equilibrium policy as a competitive equilibrium in which agents in their second period of life determine the economy's rates of labor income tax and money supply growth on a period-by-period basis. A numerical example shows that equilibrium policy can be unique and either Pareto undominated or dominated. Since agents are assumed to behave as if their policy choices have no intertemporal effects, the existence of undominated equilibrium policies is only consistent with financing relatively small levels of government expenditure.

## **1. Introduction**

Dynamic general equilibrium models provide a useful setting for analyzing the determination and welfare effects of macroeconomic policy. Since it is often the case that policy reflects the preferences of agents above a certain age (perhaps because older citizens are more likely to vote than are younger citizens) and the welfare effects of policy vary across different age groups, the overlapping generations economy represents an environment that is well suited to carrying out such studies. As an example of this type of analysis, this paper offers an equilibrium concept and investigates its welfare properties for a two-period-lived overlapping generations monetary economy in which the determination of macroeconomic policy is the responsibility of private agents in their second period of life.

In this equilibrium, what I refer to as an equilibrium policy, the old agents alive at each date  $t \geq 1$  solve a policy choice problem in which they choose the time  $t$  rates of labor income tax and money supply growth to maximize their time  $t$  utility subject to satisfying the time  $t$  government budget constraint. After showing that all time  $t$  old make the same policy choices, I illustrate by means of a numerical example the existence of both Pareto undominated as well as Pareto dominated stationary equilibrium policies. While the latter result is certainly of interest, it is the former which represents the primary result of the paper.

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Pareto undominated equilibrium policies exist despite the presence of two features that typically would be expected to preclude them from occurring. First, old agents act only in their self-interest in their role as “policymakers.” Therefore, they ignore the effects that their policy choices have on the welfare of young agents. Second, old agents are assumed to be “dynamically naive.” Specifically, they are assumed to behave as if their policy choices have no intertemporal effects despite the fact that in equilibrium this is not the case.<sup>1</sup> Thus, the existence of undominated equilibrium policies is, at a minimum, unexpected.

Within the context of an agent solving a policy choice problem, dynamic naivete is a reasonable assumption to make. First, in order to take seriously the idea that policy is determined on a period-by-period basis, at each date agents who solve the policy choice problem must only be able to determine policy for that period; policy choices for subsequent periods are beyond their control. Second, since it is agents in their final period of life who solve the policy choice problem, for them the economy (effectively) exists for only one period; what occurs after the current period ends does not matter to them. For these reasons, it follows that from the standpoint of old agents, they have no reason to consider nor means to discern that their policy choices have any intertemporal effects.

The concept of a competitive equilibrium in which each agent solves a policy choice problem has its roots in, but is distinct from, both the literature on dynamic macroeconomic policy and that on voting. First, like the former, an equilibrium policy considers the determination of macroeconomic policy instruments in a dynamic setting. However, because it is private agents rather than the government that chooses policy, an equilibrium policy can usefully be thought of as an application of the tenets of voting theory to a dynamic macroeconomic setting.<sup>2</sup> Indeed, as in much of the voting literature, the policy choice problem assumes that private agents choose their welfare maximizing value of the economy’s policy instruments subject to satisfying a government budget constraint.

Second, the usual approach in the macroeconomic policy literature is to assume that private agents and the government play a dynamic game in which the government is a dominant player.<sup>3</sup> Here, as in much of the voting literature, it is assumed that private agents take the policy choices of all other

<sup>1</sup>They affect the current period intertemporal price.

<sup>2</sup>Examples of static voting models include Borchering and Deacon (1972), Klevorick and Kramer (1973), Denzau and Mackay (1976), and Romer (1975). Dynamic examples include Hu (1982), Sjoblom (1985), Verbon (1987), Boadway and Wildasin (1989a, b), and Tabellini (1990, 1991).

<sup>3</sup>Sargent (1987), Blackburn and Christensen (1989), Blanchard and Fischer (1989), and Persson and Tabellini (1990) provide numerous examples and references.

agents as given. Thus each agent plays a (dynamic) Nash game, rather than a Stackelberg game, with all other agents. This assumption is a natural one to make in the present context since no one agent can reasonably be viewed as being a dominant player vis-a-vis all other agents. One consequence of assuming a Nash game is that the policy choice problem proves to be sequential. Hence, despite the presence of fiat money which Lucas and Stokey (1983) show can preclude time consistency, an equilibrium policy is time consistent.<sup>4</sup>

Third, the past decade has witnessed an increasing number of examples of dynamic models of voting, especially in studies of social security. Beginning with Hu (1982), a number of researchers have constructed dynamic economies where agents directly vote for the social security tax rate.<sup>5</sup> Given the complexity of these models, due in part to the implications of dynamics on the voting problem, in most of these cases expectations have not been assumed to be rational.<sup>6</sup> Rather than proceed in this fashion, I assume instead that agents possess perfect foresight. However, as the trade-offs between modeling expectations and modeling other important features of dynamic voting (for example, revoting opportunities) suggest, in order to assume perfect foresight, certain desirable features elsewhere must be weakened. For the model of this paper, it is the restriction that only agents in their second period of life solve a policy choice problem.

The absence of revoting opportunities implies that the model of this paper is not one of "voting" in the sense that this terminology is usually used. Nevertheless, it does represent a simple way to model the effects on policy when older citizens are more likely to vote than are younger citizens (as is the case in the U.S.). In particular, when this situation occurs, older citizens tend to have a disproportionate influence on policy determination and hence policy is more likely to reflect these agents' preferences. This is what occurs in an equilibrium policy.

In the next section I describe the overlapping generations economy and define both a competitive equilibrium and an undominated competitive

<sup>4</sup>For two early examples of the problem of time inconsistency, see Kydland and Prescott (1977) and Calvo (1978). Their work has led to a growing literature on mechanisms designed to overcome time inconsistency. Recent examples include Chari and Kehoe (1990, 1993), Chari, Kehoe, and Prescott (1989), Rogoff (1989), and Stokey (1989). Blackburn and Christensen (1989) and Persson and Tabellini (1990) provide extensive reviews of the literature.

<sup>5</sup>Other examples include Sjoblom (1985), Verbon (1987), Boadway and Wildasin (1989a, b), and Tabellini (1990).

<sup>6</sup>Hu (1982) represents next period's expected voting outcome by a function of the current period's vote and a random variable while Verbon (1987) uses a geometrically declining function of current and past voting decisions. In contrast, Boadway and Wildasin (1989b) assume that expectations are independent of the current period decision and that agents have perfect foresight.

equilibrium. The definitions of the policy choice problem, dynamic naivete, and an equilibrium policy appear in Section 3. Section 4 provides a numerical example which illustrates that for each of two levels of government revenue, there exists a unique equilibrium policy, one of which is an undominated equilibrium, the other of which is a dominated equilibrium. Section 5 discusses the circumstances under which undominated stationary equilibrium policies are likely to arise. The paper concludes with Section 6.

## 2. Environment and Competitive Equilibrium

The economy consists of two-period-lived overlapping generations. At each date  $t$  (where  $t = 1, 2, \dots$ ), a continuum of measure one of identical agents is born who live during periods  $t$  and  $t + 1$ .<sup>7</sup> Henceforth, let agent  $t$  be a representative member of generation  $t$ . Agent  $t$ 's time  $t$  and  $t + 1$  endowments of the economy's single non-storable consumption good,  $w_1$  and  $w_2$ , and his endowment of time  $t$  leisure,  $w_3$ , are assumed to satisfy  $w_1 \gg w_2 > 0$  and  $w_3 \gg 0$ . His preferences for time  $t$  and  $t + 1$  consumption, and time  $t$  leisure,  $c_t(t)$  and  $c_t(t + 1)$ , and  $l_t(t)$ , are represented by a smooth, strictly quasiconcave, utility function  $u[c_t(t), c_t(t + 1), l_t(t)]$  that has strictly positive first derivatives and implies that consumption at both dates and leisure are all normal goods. Finally, agent  $t$  can transform each unit of time  $t$  labor into one unit of time  $t$  good.

At  $t = 1$  there also exists a continuum of measure one of identical agents who are currently completing their lives. A representative member of this generation, agent 0, is endowed with  $w_2 > 0$  units of the time 1 consumption good and  $M(0) > 0$  units of fiat currency, and his preferences are represented by his time 1 consumption,  $c_0(1)$ .

Last, there exists a public sector that is required to raise a known quantity  $G \geq 0$  of the consumption good in every period. This expenditure is financed through a combination of labor income tax and currency issue. Thus, the time  $t$  government budget constraint satisfies

$$\tau(t)x_t(t) + p(t)[M(t) - M(t - 1)] = G, \quad t \geq 1, \quad (1)$$

where  $\tau(t)$  is the time  $t$  marginal tax rate on labor income,  $x_t(t) = w_3 - l_t(t)$  is agent  $t$ 's time  $t$  supply of labor,  $p(t)$  is the price of fiat currency in terms of the time  $t$  good, and  $M(t)$  is the time  $t$  aggregate stock of currency.

For  $t \geq 1$ , agent  $t$ 's consumer choice problem is to choose nonnegative values for  $c_t(t)$  and  $c_t(t + 1)$ , and  $l_t(t) \in [0, w_3]$ , to maximize  $u(\cdot)$  subject to  $c_t(t) + c_t(t + 1)/R(t) + [1 - \tau(t)]l_t(t) \leq w_1 + w_2/R(t) + [1 - \tau(t)]w_3$ , (2)

<sup>7</sup>This assumption is made in order to concentrate on the effects of intergenerational heterogeneity without the added complications stemming from intragenerational heterogeneity.

where both  $\tau(t) < 1$  and the time  $t$  gross real rate of return,  $R(t) > 0$ , are taken as given. Let the solution to this problem be given by the following set of smooth functions:  $x[\tau(t), R(t)]$ ,  $c_1[\tau(t), R(t)]$ , and  $c_2[\tau(t), R(t)]$ , agent  $t$ 's supply of labor, and demands for time  $t$  and time  $t + 1$  consumption, respectively. Since fiat currency is the only asset in this economy, time  $t$  saving,  $w_1 + [1 - \tau(t)]x_t(t) - c_t(t)$ , must take the form of real balances and therefore is restricted to being nonnegative. In what follows, let the smooth function  $q[\tau(t), R(t)]$  denote the time  $t$  demand for real balances.

Agent 0 solves the trivial problem maximize  $c_0(1) = p(1)M(0) + w_2$ . Thus, he inelastically supplies all of his currency to agent 1 in exchange for time 1 good.

**DEFINITION 1.** *Given  $G \geq 0$  and  $M(0) > 0$ , a competitive equilibrium (CE) consists of a pair of positive sequences  $\{1 - \tau(t), R(t)\}_{t=1}^{\infty}$  and a  $p(1) \geq 0$  such that*

$$\tau(1)x[\tau(1), R(1)] + q[\tau(1), R(1)] - p(1)M(0) = G ; \quad (3a)$$

$$\tau(t)x[\tau(t), R(t)] + q[\tau(t), R(t)] - R(t - 1)q[\tau(t - 1), R(t - 1)] = G , \quad t \geq 2 ; \quad (3b)$$

$$q[\tau(t), R(t)] \geq 0, \quad t \geq 1 . \quad (3c)$$

Consider the set of allocations that corresponds to the set of CE. In general, price distortions imply that none of these allocations is Pareto optimal.<sup>8</sup> However, since I am interested in determining whether certain CE Pareto dominate all others, it is necessary to provide an analogue to Pareto optimality that is appropriate for this setting.

**DEFINITION 2.** *Let  $\bar{b} = \{1 - \bar{\tau}(t), \bar{R}(t)\}_{t=1}^{\infty}$  be a CE.  $\bar{b}$  is an undominated competitive equilibrium if and only if there does not exist another CE,  $\hat{b} = \{1 - \hat{\tau}(t), \hat{R}(t)\}_{t=1}^{\infty}$ , such that  $\hat{b}$  supports an allocation that is weakly preferred to that of  $\bar{b}$  by all agents  $t \geq 0$  and is strictly preferred to  $\bar{b}$  by at least one such agent.*

### 3. The Policy Choice Problem

In an equilibrium policy (EP), agents in their second period of life are assumed to choose that period's labor income tax rate,  $\tau(t)$ , and money supply growth rate,  $\mu(t) \equiv M(t)/M(t - 1)$ , to maximize their utility subject to satisfying the government budget constraint, Equation (1), at time  $t$ . Following the approach often found in both the static and dynamic voting literatures, I

<sup>8</sup>One exception is the special case of the stationary equilibrium  $\tau(t) = 0$ ,  $R(t) = 1$ ,  $t \geq 1$ , and  $G = 0$ . See Wallace (1980, prop. 5).

assume that each agent takes the solutions to every other agent's policy choice problem as given when solving his own policy choice problem.<sup>9</sup> In other words, when solving the time  $t$  policy choice problem, agent  $t - 1$  is assumed to play a Nash game with all other agents. Furthermore, agents in their second period of life are assumed to be dynamically naive. These assumptions have two important implications.

First, since the time  $t$  state variable is  $M(t - 1)$ , it fully summarizes the policy choices of agents  $j < t - 1$ . Furthermore, since agent  $t - 1$ 's time  $t$  budget constraint is  $c_{t-1}(t) = p(t)M(t - 1) + w_2$ , maximizing his time  $t$  utility must necessarily entail maximizing his consumption. With  $M(t - 1)$  given, this in turn is equivalent to maximizing  $p(t)$ . Second, since agent  $t - 1$ 's policy choices determine the time  $t$  rates of labor income tax and money supply growth, only agent  $t$  is directly affected by these choices. However, because agent  $t$ 's optimal behavior implies that  $\partial u / \partial c_t(t) = R(t) \partial u / \partial c_t(t + 1)$ , any intertemporal effects brought about by agent  $t - 1$ 's policy choices must arise through changes in  $R(t)$ . Since a dynamically naive agent  $t - 1$  behaves as if his choices have no intertemporal effects, it follows that he takes  $R(t)$  as given.<sup>10</sup>

To what extent is taking  $R(t)$  as given truly "dynamically naive?" In a monetary equilibrium, the definition of real balances implies that  $q[\tau(t + 1), R(t + 1)] = \mu(t + 1)R(t)q[\tau(t), R(t)]$ . Since the demand for real balances is well defined in this situation, the implicit function theorem implies that  $R(t) = \Phi[\tau(t), \tau(t + 1), \mu(t + 1), R(t + 1)]$ . After substituting for  $R(t + j)$ ,  $j \geq 1$ , this can be expressed as  $R(t) = \Phi[\tau(t), \tau(t + 1), \dots, \mu(t + 1), \mu(t + 2), \dots]$  where Nash behavior implies that agent  $t - 1$  takes the policy decisions of agents  $t \geq j$ ,  $\{\tau(j + 1), \mu(j + 1)\}_{j=t}^{\infty}$ , as given. Hence, as a consequence of equilibrium behavior, time  $t$  policy choices (specifically that of  $\tau(t)$ ) do have intertemporal effects. Since agent  $t - 1$  behaves as if this mapping were a constant, he is "dynamically naive."<sup>11</sup>

The previous discussion can be summarized in a simple way which then leads to the definition of an EP. At time  $t \geq 1$ , agent  $t - 1$  solves the following policy choice problem: given  $R(t)$  and the time  $t$  state variable  $M(t - 1)$ , choose  $\tau(t)$  and  $\mu(t)$  to maximize  $c_{t-1}(t)$  subject to the government budget constraint, Equation (1). Since the constraint implies that agent  $t - 1$ 's objective is equivalent to  $\tau(t)x[\tau(t), R(t)] + q[\tau(t), R(t)]$ , this problem is sequential and hence is time consistent.

<sup>9</sup>Examples employing this assumption include Romer (1975), Denzau and Mackay (1976), Hu (1982), Sjoblom (1985), Verbon (1987), and Boadway and Wildasin (1989a, b).

<sup>10</sup>A similar assumption appears in Hu's (1982) overlapping generations model of social security and majority voting.

<sup>11</sup>Loewy (1988) considers a similar economy in which agents incorporate the mapping  $\Phi(\cdot)$  into their policy choice problems.

**DEFINITION 3.** *An EP is a CE that for all dates  $t \geq 1$  solves agent  $t - 1$ 's policy choice problem.*

When agent  $t - 1$  solves the time  $t$  policy choice problem, he faces a nontrivial trade-off between the labor income tax and the inflation tax. Should he choose a low income tax rate, then the resultant low level of income tax revenue implies that the inflation tax must be large. The required large increase in the money supply growth rate reduces  $p(t)$ , ceteris paribus. On the other hand, should he choose a high tax rate, then even though the inflation tax is now smaller, so too is the demand for real balances. This also reduces  $p(t)$ , ceteris paribus. As will be shown below, the optimal values of the tax rate and money supply growth rate balance the marginal effects of the labor income tax and the demand for real balances on agent  $t - 1$ 's objective,  $p(t)$ .

Finally, since all members of generation  $t - 1$  have the same objective, their optimal policy choices are identical. The presence of unanimity implies that appeals to the median voter rule are unnecessary. This would remain true even if there existed limited diversity across members of a given generation. In this case, all members of generation  $t - 1$  would still seek to maximize the time  $t$  value of their fiat currency (though they now hold different amounts). After making use of the government budget constraint, they would continue to maximize (effectively) the same objective, the aggregate quantity of goods supplied by the members of generation  $t$ .

#### **4. Equilibrium Policy: An Example**

In this section, I construct an example that is used to illustrate the existence of both Pareto undominated and Pareto dominated stationary monetary EP.<sup>12</sup> Before turning to the example, I first introduce some useful notation. Let  $r[\tau(t), R(t)] = \tau(t)x[\tau(t), R(t)] + q[\tau(t), R(t)]$ . Using the young agent's budget constraint,  $c_t(t) + q(t) \leq [1 - \tau(t)]x_t(t) + w_1$ ,  $r[\tau(t), R(t)]$  is easily seen to equal  $x[\tau(t), R(t)] + w_1 - c_1[\tau(t), R(t)]$ , his excess supply of time  $t$  good. Next, let  $g(\tau, R) = r(\tau, R) - Rq(\tau, R)$ . Substituting  $r(\cdot)$  into the stationary version of the government budget constraint implies that  $g(\tau, R)$  determines the level of revenue that the government raises in a stationary CE. Finally, define  $B(G)$  to be the set of stationary CE for a given value of  $G$ .

Let  $u = \ln c_t(t) + \ln c_t(t + 1) + \ln l_t(t)$  and  $(w_1, w_2, w_3) = (6, 1, 18)$ . The value of  $M(0)$  is arbitrary as long as it is positive. Maximizing this utility subject

<sup>12</sup>Since the focus of this paper is on the welfare properties of equilibrium policies, proofs of the existence and generic finiteness of stationary monetary EP are omitted. They are available from the author upon request.

to Equation (2) yields the usual demand functions. Substituting these into Equation (3) then produces  $B(G)$ . If, for example,  $G = 1$ , then  $B(1)$  contains two nonmonetary equilibria,  $(\tau, R) = (0.189, 0.097)$  and  $(0.589, 0.149)$ , two nonproduction equilibria,  $(\tau, R) = (0.75, 0.3)$  and  $(0.7, 0.5)$ , and two continua of interior equilibria.<sup>13</sup>

In order to determine which stationary CE are also stationary EP, it is first necessary to solve agent  $t-1$ 's policy choice problem. Let  $R(t) = R$  be given. After making use of the government budget constraint and noting that once  $\tau(t)$  and hence  $p(t)$  are determined, the choice of  $\mu(t)$  is dictated by market clearing (recall that  $M(t-1)$  is the time  $t$  state), agent  $t-1$ 's policy choice problem reduces to choosing  $\tau(t)$  to maximize  $r[\tau(t), R]$ . The properties of  $r(\cdot)$  imply that this problem possesses a solution,  $\tau^*(t)$ , where  $(\tau^*(t), R)$  necessarily satisfies

$$\tau^*(t) = \tau_{nm} \quad , \quad \text{if } q(\tau^*(t), R) = 0 \quad , \quad (4)$$

$$\partial r(\tau^*(t), R) / \partial \tau = 0 \quad , \quad \text{if } q(\tau^*(t), R) > 0 \quad , \quad (5)$$

and where  $\tau_{nm}$  is the tax rate that maximizes the level of government revenue that can be raised in a nonmonetary equilibrium. Note that if  $(\tau^*(t), R)$  satisfies Equation (5), then the definition of  $r(\cdot)$  implies that  $\tau^*(t) \partial x / \partial \tau + x[\tau^*(t), R] + \partial q / \partial \tau = 0$ . Thus, the optimal choice of  $\tau(t)$  trades off the change in labor income tax revenue against the change in the base (and since  $R$  is given, the level) of the inflation tax. Furthermore, since saving falls with an increase in the labor income tax rate, agent  $t-1$  finds it optimal to set  $\tau(t)$  at a value below that which maximizes revenue from the labor income tax.

Consider the implications of Equations (4) and (5) for the example described above. First, note that  $\tau_{nm} = 0.423$ . Next, let  $\tau(R)$  be such that  $\partial r[\tau(R), R] / \partial \tau = 0$ . Direct calculation then shows that  $\tau(R) = 1 - [(6 + R^{-1}) / 18]^{1/2}$ . Since the choice of  $t$  is irrelevant in a stationary equilibrium, I can show that the solution to every old agent's policy choice problem is given by the piecewise smooth function

$$\tau^*(t) = \tau^*(R) = \begin{cases} 0.423 & R \in (0, 0.104] \\ \tau(R) & R \in [0.104, \infty) \end{cases} \quad . \quad (6)$$

<sup>13</sup>For  $\tau \in \{[-0.567, 0.189) \cup (0.589, 0.75) \cup (0.7, 0.787]\}$ , these continua contain the points  $(\tau, R_1)$  and  $(\tau, R_2)$  where  $0 < R_1 < R_2$  are the two roots of the following quadratic in  $R$ :  $6(1 - \tau)[3(1 - \tau) + 1]R^2 + [18(1 - \tau)^2 - 45(1 - \tau) + 6]R - \tau = 0$ . For  $\tau \in \{(0.189, 0.589) \cup (0.75, 0.7)\}$ , the presence of corner solutions implies that these continua only contain the points  $(\tau, R_2)$ .

Hence, any  $(\tau, R) \in B(G)$  such that  $\tau = \tau^*(R)$  is a stationary EP. In particular, there exists a stationary monetary EP for all  $G \in [0, 5.141]$ . In addition, if  $G \in [0, 1.495) \cup \{5.141\}$ , then this equilibrium is unique.<sup>14</sup>

By Definition 2, the set of undominated stationary CE corresponds to the solutions to the following problem: for each  $\lambda \in [0, 1]$ , choose  $(\tau, R) \in B(G)$  to maximize  $\lambda c_2(\tau, R) + (1 - \lambda)v(\tau, R)$  where  $v(\tau, R)$  is the indirect utility of the representative young agent. Given  $G$ , maximizing  $c_2(\cdot)$  (that is,  $\lambda = 1$ ) is equivalent to minimizing of the sum of first-period consumption plus leisure. On the other hand, maximizing  $v(\cdot)$  (that is,  $\lambda = 0$ ) places positive weight on both consumptions and on leisure. Hence, the stationary CE which maximizes  $v(\cdot)$  has a lower level of second-period consumption and higher levels of first-period consumption and leisure than does the one which maximizes  $c_2(\cdot)$ . Furthermore, since both consumptions and leisure are gross substitutes in this example, setting  $\lambda = 0$  yields a lower labor income tax rate and a lower rate of return than does setting  $\lambda = 1$ . Indeed, given  $G$ , both  $\tau$  and  $R$  monotonically increase as  $\lambda$  increases from 0 to 1.

Let  $G = 1$ . The set of undominated equilibria are those elements of  $B(1)$  that lie along the continuum bounded at one end by  $(\tau, R) = (0.080, 0.964)$  and at the other by  $(\tau, R) = (0.423, 1.487)$ . Since  $G < 1.495$ , there exists a unique stationary monetary EP,  $(\tau(R), R) = (0.390, 1.433)$ . As this equilibrium belongs to the required continuum, it is an undominated stationary CE.

Next, let  $G = 5.141$ . In this case, the set of undominated stationary CE lie along a continuum in  $B(5.141)$  bounded by  $(\tau, R) = (0.390, 0.499)$  and  $(\tau, R) = (0.423, 0.522)$ . However, since the economy's unique stationary monetary EP,  $(\tau(R), R) = (0.309, 0.384)$ , does not belong to this continuum, no undominated stationary EP exists.

Given that old agents act in their own self-interest and are dynamically naive, it is not particularly surprising to find an example in which the stationary EP is a dominated equilibrium. Conversely, it is quite surprising that despite these features, there nevertheless exists an example in which the stationary EP is an undominated equilibrium. This leads one to ask under what circumstances are stationary EP undominated equilibria?

## 5. Undominated Stationary EP

In my model, all saving is in the form of real balances. Assuming that stationary monetary equilibria exist, it is clear that valued currency is a necessary condition for an undominated equilibrium. This implies that in searching for conditions under which undominated stationary EP exist, at-

<sup>14</sup>Otherwise there exist two such equilibria. There also exists a continuum of stationary nonmonetary equilibria for  $G = 1.507$  which correspond to the upper branch of Equation (6).

tention need only be focused on stationary monetary EP. Hence, in terms of the example of the previous section, all relevant stationary EP satisfy  $\tau^*(R) = \tau(R) = 1 - [(6 + R^{-1})/18]^{1/2}$ . Additionally, for  $R \geq 0.384$ , these equilibria are such that they lie along the efficient side of the EP government revenue function,  $g[\tau(R), R]$ ,<sup>15</sup> a necessary, but not sufficient, condition for the existence of an undominated equilibrium.

Consider next how the allocation in a stationary monetary EP changes as one varies the equilibrium rate of return. To simplify the discussion, yet not sacrifice too much generality in doing so, I again focus on the example of the previous section. The above expression for  $\tau(R)$  shows that it is strictly increasing in  $R$ . This in turn implies that  $dc_1/dR < 0 < dc_2/dR$  and  $dl/dR < 0$  in any stationary EP which satisfies the above necessary condition for an undominated equilibrium. Suppose first that  $R$  is "large." It then follows that the level of  $c_2$  is large, and the levels of  $c_1$  and  $l$  are small relative to their values in stationary EP with lower rates of return. As  $R$  falls, this disparity between  $c_1$ ,  $c_2$ , and  $l$  is eventually eliminated and then is reversed.

Let  $R$  again be "large." Since real balances are necessarily bounded above, it follows that if  $R > 1$ , then seigniorage revenue,  $(1 - R)q[\tau, R]$ , is negative in such a stationary CE. Furthermore, this problem is exacerbated the lower is  $\tau$ . On the other hand, though the intertemporal effect tends to increase labor supply when the rate of return is high, the high tax rate needed to limit seigniorage losses implies that labor income tax revenues are limited. This implies that stationary CE with "large" values of  $R$  typically finance "small" levels of government expenditure. Thus, the same must be true of stationary EP where  $R$  is large.

Suppose, then, that  $(\tau(R), R)$  is a stationary EP in which  $R$  is "large" and hence,  $G$  is "small." Normal goods implies that if both  $R$  and  $\tau = \tau(R)$  are large relative to their values elsewhere in  $B(G)$  (as they are here), then so too is the level of  $c_2$  relative to the levels of  $c_1$  and  $l$ . However, it is because  $G$  is small and therefore seigniorage revenue can be negative (that is,  $R > 1$ ) that such an allocation closely approximates what a planner seeking to maximize  $c_2(\cdot)$  would choose. It follows that when  $G$  is small, the stationary EP is typically an undominated equilibrium.

Suppose next that the level of  $R$  decreases. Since the economy is on the efficient side of the EP government revenue function, it follows that the level of government revenue raised in an EP increases. Since the decline in  $R$  reduces and then eliminates the disparity between the level of  $c_2$  and the levels of  $c_1$  and  $l$ , the allocation supported as a stationary EP begins to approximate the one which a planner seeking to maximize  $v(\cdot)$  would choose.

<sup>15</sup>Since  $R = 1/\pi$  in a stationary equilibrium, where  $\pi$  is the gross inflation rate, it follows that the efficient side of this function satisfies  $dg/dR < 0$ .

Hence, for levels of  $G$  which are not too large, an equilibrium policy continues to be an undominated equilibrium.

Last, suppose that  $R$  is approximately equal to its minimum value such that the economy remains on the efficient side of the EP government revenue function. Hence,  $g[\tau(R), R]$  is near its maximum value. Normal goods implies that if both  $R$  and  $\tau = \tau(R)$  are small relative to their values elsewhere in  $B(G)$  (as they are here), then so too is the level of  $c_2$  relative to the levels of  $c_1$  and  $l$ . In many cases, this disparity is great enough that the stationary EP no longer supports an allocation that approximates even the one which maximizes  $v(\cdot)$ . Therefore, despite the EP lying on the efficient side of the government revenue function, the levels of both  $c_2$  and  $v$  can be increased by increasing both  $\tau$  and  $R$  in a manner such that  $g(\tau, R) = G$ .

## **6. Conclusion**

This paper considers a two-period-lived overlapping generations monetary economy in which agents in their second period of life determine the economy's rates of labor income tax and money supply growth on a period-by-period basis. This decision, what I define as the agent's policy choice problem, assumes that each agent is dynamically naive, that is he behaves as if his choices have no intertemporal effects, and plays a Nash game with all other agents. By means of a numerical example, I show that for each of two levels of government revenue, there exists a unique equilibrium policy, one of which is an undominated CE, the other of which is a dominated CE.

Dynamic naivete and the Nash game of this model make operational the idea that agents in the final period of life determine policy on a period-by-period basis. In particular, they make explicit the reasonable criteria that an old agent setting policy at time  $t$  can only determine policy for that period and that, for him, the economy effectively exists for one period. Adding to this the fact that agents act in their own self-interest when solving their policy choice problem suggests that EP should typically be dominated equilibria. Consequently, the existence of undominated EP is rather surprising. Since these EP exist as long as the rate of return is not too small, they are generally consistent with financing levels of government expenditure that are bounded away from the maximum level that can be financed in a stationary EP. Therefore, dynamically naive policymakers are more likely to make socially optimal choices the smaller is the size of the economy's public sector.

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