Factor Substitution, Price Elasticity of Factor Demand and Returns to Scale in Police Production: Evidence from Michigan*

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I. Introduction

Budget problems facing many state and local governments, together with declining public sector subsidies, have forced these governments to search for more efficient ways of providing services such as public safety and public health. The provision of public safety, like many other service industries, is labor intensive with personnel cost usually accounting for between 50 and 70 percent of total cost. The principal labor input is police officers. The relatively large increases in police wages (partly aided by arbitration) thus puts increased pressures on city budgets. Apparently, efforts at reducing cost of providing public safety have included attempts at factor substitution. For example, the last decade has seen an increasing use of civilian employees, mostly in administration, so that the ratio of police officers to civilians has declined.

In addition, there has been a decline in the use of sidewalk beats by some police departments and an increase in the use of patrol cars for surveillance. Evidently there is a possible output as well as a substitution effect in the demand for these factors.

The operational questions raised by these observations are: to what extent is it possible to substitute civilian employees or patrol cars for police personnel without loss of efficiency? Are civilian employees and capital inputs substitutes or complements to police personnel in production? How price elastic is the demand for these factors in the production of police services? Will consolidation of police departments in a metropolitan area yield substantial reductions in unit costs of police services? Are there economies of scope in police production? In this paper we attempt to answer these questions by calculating elasticities of factor substitution, price elasticities of factor demand, economies of scope, and returns to scale in police production using data from municipal police departments in the state of Michigan.

In the past few years a number of authors have estimated production functions for law

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enforcement. We briefly review the findings of these authors. Most of the attempts were part of a simultaneous system in determining the supply of offenses so that only scant attention was paid to the substitution among inputs [9; 11; 12]. Infact, some of these even used only one input, thus eliminating the possibility of input substitutability.

A few attempts have been made, however, to estimate single-equation production functions for police departments. For example, Chapman, Hirsch and Sonenblum [4] estimate a rather traditional production function for the Los Angeles Police department between 1956 and 1970. All outputs are collapsed into one aggregate which is then regressed on 4 types of inputs—motorcycle teams, field officers, nonfield officers and civilian employees. They find strongly increasing returns to scale—often a two to four percent output response to a one percent change in input usage. However, by assuming fixed capital-labor ratios within these teams, they could not measure substitution between capital and labor. Chapman, Hirsch, and Sonenblum were more concerned with scale economies than with substitutability between inputs.

Votey and Phillips [24] and Phillips and Votey [20] estimated production function for four property crimes for the U.S. and California respectively. While they employed two inputs-capital and police-to estimate a separate production function for each of the four outputs, they did not measure elasticity of substitution between these inputs.

Certain characteristics common to all these studies can be identified. First, they adopt the Cobb-Douglas functional form which has some restrictive properties. Among these is the characteristic of unit elasticity of substitution between any pair of inputs. At best, unit elasticity of substitution should be maintained as a testable hypothesis.

Second, some of the studies utilize a single output aggregate [4; 25]. If the results of such aggregate studies are to be used for decision purposes, it is desirable to test for the existence of a consistent aggregate indices of police output. Even in studies that recognize that police departments produce multiple outputs, the authors still maintain the hypothesis of nonjoint outputs [24; 26]. The assumption of nonjointedness makes it possible to estimate a separate production function for each of the outputs produced by the police. However, Darrough and Heineke [7] have shown that police production is joint. Estimating a separate production function for each of the jointly produced outputs will result in biased parameter estimates.

The development of flexible functional forms[1] such as the translog production function [6], the generalized Cobb-Douglas production [8], and duality theorems has made it possible to estimate multiproduct production (cost) functions and to test the structure of the underlying production technology. There has been very limited application of flexible functional forms to the area of police production. Darrough and Heineke [7] estimated a multiple output translog cost function for a sample of 30 medium sized U.S. city police departments, and tested various hypotheses concerning the production structure of law enforcement agencies, such as nonjointness of production, and constant returns to scale. Their cost function, however, included one input price—police wage. Thus it was not possible to estimate elasticities of factor substitution. Moreover, if important inputs in the production process are excluded, the parameter estimates may be biased unless the excluded inputs are highly correlated with the included inputs.

Phillips [19] uses a generalized constant difference of substitution in direct production function (CDE) developed by Hanoch [13] to estimate factor demands for 5 city police departments in California. He reported large substitution possibilities with considerable variation among the cities

1. In general, "flexible" functional forms are second order approximations to the primal or dual objective function in optimization problems.

in the sample. For the case of Long Beach, San Diego and Oakland, Phillips found substitution elasticities of 2.08 for police-vehicle, police-civilian employees, and vehicle-civilian employee substitution, while for San Francisco, the substitution elasticities for all pairs of inputs is 1.12. For Los Angeles, the elasticities of substitution are .263 for police-vehicle and police-civilian and .580 for civilian-vehicle substitution. While Phillips was able to calculate elasticities of substitution, the use of a CDE production function implied some constancy of elasticities of substitution among the inputs.

This paper investigates the characteristics of police production using data from municipal police departments in Michigan. It is by studying the characteristics of police production functions with different data sets and modelling that generalizations can be made about the functional structure of police production.

Because a central concern of this paper is to study the extent of substitutability among inputs, it is important to choose a functional form which places no a priori restrictions on substitution possibilities. The translog function does not have such a restrictive property [6]. Our approach then is to use data from municipal police departments in Michigan to estimate a multiple output translog cost function and employ duality theorems to infer the implied substitution elasticities. In addition, we also test explicitly for several of the implicit assumptions made in earlier studies.

The rest of the paper is organized as follows: Section II specifies the theoretical and econometric model, section III discusses the data, section IV presents the empirical results, section V addresses the issue of returns to scale and economies of scope while section VI concludes the paper.

II. The Model

The economic view of police department behaviour, similar to that used by Darrough and Heineke [7], is to presume that police departments act to minimize the cost of providing an acceptable level of output subject to a technology constraint. Stated differently, given input prices, the police department is assumed to choose an input combination that minimizes the cost of production. We assume the existence of a strictly concave differentiable police production function. Diewert [8] has shown that there exists a cost function in input prices and outputs that is dual to the underlying production technology.[2] We approximate the unknown cost function with a multiple-output- multiple-input translog cost function [6].

$$\ln C=\alpha_0 + \sum_i \alpha_i \ln Y_i + \sum_j \beta_j \ln W_j + \Big(\sum_i \sum_j \alpha_{ij} \ln Y_i \ln Y_j\Big)/2 + \Big(\sum_j \sum_k \beta_{jk} \ln W_j \ln W_k\Big)/2 + \sum_i \sum_j \gamma_{ij} \ln Y_i \ln W_j, \quad (1)$$

where C = total cost; Y_i, Y_j is a set of output; W_j, W_k are the prices of inputs j and k; and α_i, β_j, α_{ij}, β_{jk} and γ_{ij} are parameters. The translog cost function is a second-order approximation to an arbitrary cost function and in particular places no restrictions on elasticities of substitution between inputs and allows returns to scale to vary with the level of output. This allows us to empirically estimate substitution possibilities. While equation (1) involves a large number of parameters, it can be estimated within a systems framework given sufficient observations. Also,

2. We use the cost function since it is more appropriate to take prices as exogenous than quantities

the efficiency of the resulting estimates are enhanced if the relevant share equations are estimated simultaneously with the cost function. From Shepherd's Lemma the derived demand functions are found by differentiating the translog cost function with respect to the input prices. The share equations are given by $S_j = \partial \ln C/\partial \ln W_j = W_j X_j/C$, or

$$S_j = \beta_j + \sum_i \gamma_{ij} \ln Y_i + \sum_k \beta_{jk} \ln W_k \qquad \text{for } j = 1, \ldots, n. \tag{2}$$

The translog cost function must be homogenous of degree one in input prices, and that the second-order coefficients must be symmetric. This implies the following parameter restrictions [6]:

$$\sum_j \beta_j = 1, \quad \sum_j \gamma_{ij} = \sum_j \beta_{jk} = \sum_k \beta_{jk} = 0, \quad \alpha_{ij} = \alpha_{ji}, \quad \beta_{jk} = \beta_{kj} \text{ and } \gamma_{ij} = \gamma_{ji}.$$

Finally, if the elasticities of substitution between all factors are equal to 1 (so that the cost function is a Cobb-Douglas cost function), additional parameter restrictions, $\beta_{jk} = 0 = \gamma_{ij}$ are implied.

We represent the output of police departments by the number of arrests for each of eight Federal Bureau Investigation (FBI) index crimes: murder and negligent manslaughter, rape, aggravated assault, robbery (rob), burglary (burg), larceny (larc), motor vehicle theft (mvth), and arson (ars). We aggregated the arrests for murder, negligent manslaughter, rape, and aggravated assault into one output, personal crimes (pers). This was done partly to reduce the number of departments which did not report any output on murder. Thus failure to aggregate would mean excluding those departments and this might introduce the possibility of sample bias. The police also produce other rather mundane outputs such as directing traffic, investigating accidents, providing emergency first aid and quieting family squabbles. Following [7], we assume that these non-arrest outputs are proportional to the size of the city (population) in which the police department is located. We group all such non-arrest outputs together and use the population of the city served as a proxy to measure the output of these activities. Thus we have seven measures of output-six of them arrest-related and one representing all non-arrest output-in our cost equation.

There are, however, a few problems associated with using the number of arrests as our measure of police output. First, it does not consider no arrest outputs. Second, the number of arrests depends in part, on the number crimes reported. In view of the fact crimes are underreported, the number of arrests may underestimate police output. On the other hand, given the possibilities of wrongful or multiple arrests, the number of arrests may overestimate police output. These problems of output measure implies that one should exercise caution when interpreting the results of work on police production functions.

The inputs used by police departments are two types of labor: sworn police officers and civilian employees, and an assortment of capital inputs such as motor cycles, radars and police squad cars. Since prices could not be obtained for all of the disaggregated capital inputs, we used the police squad car as an index of capital inputs.

With seven outputs and three inputs, our multi-output-multi-input translog cost function is given as:

$$\begin{aligned} \ln C = {} & \alpha_0 + \sum_{i=1}^{7} \alpha_i \ln Y_i + \sum_{j=1}^{3} \beta_j \ln W_j + \Big(\sum_i^7 \sum_j^7 \alpha_{ij} \ln Y_i \ln Y_j\Big)/2 \\ & + \Big(\sum_{j=}^3 \sum_k^3 \beta_{jk} \ln W_j \ln W_k\Big)/2 + \sum_i^7 \sum_j^3 \gamma_{ij} \ln Y_i \ln W_j, \end{aligned} \tag{3}$$

Table I. Descriptive Statistics of Cost Function Variables

Variable	Sample Mean	Standard Error
Total Cost	4,251,796.95	1,337,066.71
Police Wage	37,318.04	906.86
Civilian Wage	15,554.46	156.19
Price of Capital	11,439.00	6.77
Robbery	36.88	17.86
Burglary	71.60	22.03
Larceny	206.92	32.24
Motor Vehicle Theft	23.13	7.90
Arson	4.90	1.83
Personal Crime	80.83	29.98
Population	36,765.22	6,223.38
Police Share	0.7984	0.0053
Civilian Share	0.0660	0.0027
Capital Share	0.1356	0.0046

Notes: The number of observations is 260.
All monetary variables are in real values.

where $\alpha_{ij} = \alpha_{ji}$, $\beta_{jk} = \beta_{kj}$, $\Sigma\beta_j = 1$ and $\Sigma\beta_{jk} = \Sigma\gamma_{ij} = 0$. The corresponding cost share equations are:

$$S_j = \beta_j + \sum_{k}^{3} \beta_{jk} \ln W_j \ln W_k + \sum_{i}^{7} \gamma_{ij} \ln Y_i + \nu, \qquad \text{for } j = 1, \ldots, 3. \tag{4}$$

Relative shares thus depend on factor prices and output. We delete one share equation—the share of capital—to avoid singularity since the share equations must add up to unity. Our final econometric model, then, consists of the cost function (3) plus 2 share equations (4). With symmetry and linear homogeneity imposed, the number of parameters to be estimated comes to 55.

III. Data

The model is estimated using cross-sectional data from 130 Michigan State municipal police departments serving cities with populations of 5,000 or more for the years 1984 and 1985. All the data on crime and population were obtained from Michigan Department of State Police. Total expenditure for law enforcement as well as the portion going to compensation of police and civilian personnel are published annually by the Michigan Department of Commerce. Police wage was calculated as the average (unweighted) salary of a police officer in a department. It was not possible to disaggregate by rank.

Data on salaries of civilian employees in police departments were not available. However, most of these civilian employees perform clerical duties. Assuming that the police department operates in a competitive labor market, the wages of its civilian employees will approximate those of employees with similar skills in non-police departments. Hence the average wage of clerical workers in the city in which the police department is located was used as a proxy for wages of civilian employees. Data on average hourly wages for this occupation were available from the Michigan Municipal League. Using a standard 40-hour work week and a 50-week work year, we converted the hourly wages to annual equivalents. To make the civilian wage comparable to the police wage which included fringe benefits, we adjusted the former figures upward by 20 percent.[3]

The police patrol car is used as an index of capital. The price of an automobile comparably equipped to withstand the demands of patrol is thus used as the price of capital. Following [19], we choose the price of a comparably equipped four-door Oldsmobile Delta 88 as the price of capital. The information was obtained from *Automotive News*, published annually [2].

Employment data used to calculate the factor cost shares are available in the annual volumes of the Michigan Department of State Police. All monetary variables are deflated by the Consumer Price Index for the North East Central United States to take account of the temporal price changes. Table I presents some descriptive statistics of our data. It shows that about 80 percent of total expenditures on law enforcement went to sworn police officers, with only 7 percent going to civilian employees in police departments and 14 percent to capital.

IV. Estimation and Results

There were 260 observations available for estimating each equation in the system. In order to provide a stochastic framework for equations (3) and (4), we append classical additive disturbances to each of the equations. We assume no autocorrelation of error terms within, but account for contemporaneous correlation across equations. We make no other assumptions about the disturbances other than they be uncorrelated with right hand side variables in each equation. In the computations, we use the iterative Zellner efficient (IZEF) method. We choose this procedure for a variety of reasons. First, based on our assumption of no autocorrelation within equations, the IZEF method produces parameter estimates which are full information maximum likelihood estimates [14]. Second, the estimates produced by the IZEF procedure are invariant to which equation is deleted [27].

Parameter estimates are shown in Table II. The estimates reported in column two contain no restrictions other than symmetry and homogeneity of C in input prices. Given the cross sectional nature of the data, the model fits quite well with adjusted R^2s of .965 for the cost function and .217 and .118 for the share equations for police and civilian, respectively. All three first-order input price coefficients are positive and highly significant at $\alpha = .01$, indicating that the cost function is increasing in input prices. Similarly, the second-order input coefficients are significantly different from zero. Considering our output coefficients, we find that most of them are not significant. We suspect that the apparent insignificance is primarily attributable to collinearity among some of the output. For example, burglary will have a high correlation with robbery and larceny.

In columns 3 and 4 are parameter estimates for the cases of single input, and Cobb Douglas cost function, respectively. The model with only one input (column 3), sworn police, was estimated primarily to contrast with earlier work [7]. Darrough and Heineke [7] employed only one input in their estimation of a multiproduct translog cost function. They justified this approach by the fact that most of the expenditure on law enforcement is taken by police wages and salaries. This restriction reduces the number of parameters which must be directly estimated to 36. The first and second-order output coefficients are similar to those in the full (unrestricted) model, with most remaining insignificant.

3. This figure was suggested to the second author by the Labor Department's Atlanta Office.

Table II. Parameter Estimates of Translog Cost Function

Parameters	Symmetry & Linear Homogeneity Model	Model with Only Police Input	Cobb-Douglas Model
α_0	−0.8478	−5.4367	−4.1331
	(7.6714)	(8.0975)	(0.4093)
α_1	−0.1651	0.3168	0.0902
	(0.7443)	(0.7867)	(0.0340)
α_2	−0.4459	0.4987	0.0696
	(1.1048)	(1.1634)	(0.0567)
α_3	−0.9332	0.1553	−0.0690
	(0.7748)	(0.8153)	(0.0354)
α_4	1.0815	0.5855	0.0964
	(1.0267)	(1.0852)	(0.0456)
α_5	−1.0837	−1.9671	−0.0106
	(0.8514)	(0.8988)	(0.0447)
α_6	0.9315	1.2066	0.7876
	(1.8729)	(1.9734)	(0.0528)
α_7	0.4214	0.6493	0.0325
	(0.5919)	(0.6259)	(0.0315)
β_1	0.5839	1.0000	0.8074
	(0.1107)	—	(0.0067)
β_2	0.0342	—	0.0623
	(0.0609)	—	(0.0033)
β_3	0.3551	—	0.1303
	(0.1100)	—	(0.0061)
α_{11}	0.0216	0.0305	—
	(0.1033)	(0.1088)	—
α_{12}	−0.1901	−0.1136	—
	(0.0739)	(0.0782)	—
α_{13}	−0.0838	−0.0281	—
	(0.0613)	(0.0649)	—
α_{14}	0.0427	−0.0270	—
	(0.0888)	(0.0939)	—
α_{15}	0.0772	0.1051	—
	(0.0715)	(0.0748)	—
α_{16}	0.0866	−0.0055	—
	(0.0880)	(0.0931)	—
α_{17}	0.0377	0.0182	—
	(0.0625)	(0.0661)	—
α_{22}	0.0728	−0.0007	—
	(0.2532)	(0.2642)	—
α_{23}	0.1462	0.1483	—
	(0.0894)	(0.0946)	—
α_{24}	−0.1427	−0.0412	—
	(0.1217)	(0.1289)	—

Table II. Continued

Parameters	Symmetry & Linear Homogeneity Model	Model with Only Police Input	Cobb-Douglas Model
α_{25}	−0.0435	0.0199	—
	(0.1360)	(0.1434)	—
α_{26}	0.0452	−0.0854	—
	(0.1559)	(0.1636)	—
α_{27}	−0.0866	−0.0434	—
	(0.1108)	(0.1162)	—
α_{33}	−0.1631	−0.1060	—
	(0.1049)	(0.1101)	—
α_{34}	0.0265	0.0718	—
	(0.0878)	(0.0931)	—
α_{35}	−0.0922	−0.0578	—
	(0.0648)	(0.0686)	—
α_{36}	0.1568	−0.0124	—
	(0.1089)	(0.1142)	—
α_{37}	0.0167	−0.2311	—
	(0.0585)	(0.0618)	—
α_{44}	0.2730	0.1506	—
	(0.1315)	(0.1390)	—
α_{45}	−0.0808	−0.0617	—
	(0.1056)	(0.1113)	—
α_{46}	−0.1110	−0.0818	—
	(0.1295)	(0.1371)	—
α_{47}	−0.0014	−0.0065	—
	(0.0634)	(0.0672)	—
α_{55}	0.1157	−0.0871	—
	(0.1329)	(0.1388)	—
α_{56}	0.1280	0.2301	—
	(0.1071)	(0.1129)	—
α_{57}	0.0514	0.0767	—
	(0.0826)	(0.0861)	—
α_{66}	−0.1406	−0.0568	—
	(0.2388)	(0.2510)	—
α_{67}	−0.0492	−0.0737	—
	(0.0765)	(0.0809)	—
α_{77}	0.0672	0.0655	—
	(0.0823)	(0.0872)	—
β_{11}	0.0877	—	—
	(0.019)	—	—
β_{12}	−0.0051	—	—
	(0.0023)	—	—
β_{13}	−0.0826	—	—
	(0.0183)	—	—
β_{22}	0.0316	—	—
	(0.0220)	—	—

Table II. Continued

Parameters	Symmetry & Linear Homogeneity Model	Model with Only Police Input	Cobb-Douglas Model
β_{23}	−0.0265	—	—
	(0.0223)	—	—
β_{33}	0.1091	—	—
	(0.0284)	—	—
γ_{11}	0.0089	—	—
	(0.0092)	—	—
γ_{12}	−0.0154	—	—
	(0.0153)	—	—
γ_{13}	−0.0255	—	—
	(0.0095)	—	—
γ_{14}	0.0094	—	—
	(0.0123)	—	—
γ_{15}	0.0047	—	—
	(0.0123)	—	—
γ_{16}	0.0237	—	—
	(0.0143)	—	—
γ_{17}	−0.0050	—	—
	(0.0085)	—	—
γ_{21}	−0.0019	—	—
	(0.0048)	—	—
γ_{22}	0.0091	—	—
	(0.0079)	—	—
γ_{23}	0.0372	—	—
	(0.0134)	—	—
γ_{24}	−0.0029	—	—
	(0.0064)	—	—
γ_{25}	−0.0063	—	—
	(0.0065)	—	—
γ_{26}	−0.0050	—	—
	(0.0075)	—	—
γ_{27}	0.0014	—	—
	(0.0044)	—	—
γ_{31}	−0.0071	—	—
	(0.0089)	—	—
γ_{32}	0.0062	—	—
	(0.0148)	—	—
γ_{33}	−0.0054	—	—
	(0.0174)	—	—
γ_{34}	−0.0065	—	—
	(0.0019)	—	—
γ_{35}	0.0015	—	—
	(0.0139)	—	—
γ_{36}	−0.0186	—	—
	(0.0139)	—	—

Table II. Continued

Parameters	Symmetry & Linear Homogeneity Model	Model with Only Police Input	Cobb-Douglas Model
γ_{37}	0.0036	—	—
	(0.0083)	—	—
N	260	260	260

Notes: Standard errors are in parentheses.
Parameters are indexed as follows:
output: rob = 1, burg = 2, larc = 3, mvth = 4, pers = 5, pop = 6, ars = 7.
input: *WPOL* = 1, *WCIV* = 2, *PCAP* = 3.

An appropriate test of the hypothesis of equality of the full model and the restricted model with only police input is based on the likelihood ratio statistic:

$$-2\log\Lambda = N\left[\log|\Omega_r| - \log|\Omega_u|\right] \tag{5}$$

where $|\Omega_r|$ and $|\Omega_u|$ are absolute values of the determinants of the estimated error covariance matrices for the restricted and unrestricted models respectively, and N is the number of observations. This statistic is distributed as a chi-square with degrees of freedom equal to the number of imposed parameter restrictions. The calculated value of the likelihood ratio statistic is 628.371. With 16 parameter restrictions being tested, we can easily reject the model with police as the only input.

The spirit of our effort is to estimate elasticities of substitution and price elasticities of demand from cost-minimizing factor demand equations. In order to estimate substitution elasticities, however, we must first see if the underlying production function is Cobb-Douglas. Since the Cobb-Douglas production function implies unit elasticity of substitution, we do not calculate substitution elasticities if the null hypothesis that the cost function is a Cobb-Douglas cost function is not rejected (i.e., that β_{jk} are all zero). These restrictions can be tested using the likelihood ratio test defined in equation (5). The calculated statistic is 72.96. The test statistics are significant at the 5 percent level, so that the hypothesis that the corresponding production function is Cobb-Douglas can be rejected. This result colloborates earlier findings by [7].

We are now in a position to calculate the elasticities of substitution. Elasticities of substitution measures the change in a firm's demand for factor j in response to a change in the price of input i, all things equal. Allen [1] has defined the partial elasticity of substitution (APES) between inputs i and j as:

$$\sigma_{ij}^{A} = \left(\sum X_k f_k / X_j X_i\right)\left(\overline{F}_{ij} / \det F\right), \tag{6}$$

where det **F** is the determinant of the bordered Hessian matrix

$$\begin{pmatrix} 0 & F_1 & F_2 & \cdots & F_n \\ F_1 & F_{11} & \cdot & \cdots & \cdot \\ \vdots & & & & \\ F_n & \cdot & \cdot & \cdots & F_{nn} \end{pmatrix}$$

and $\overline{F}_{ij}$ is the cofactor of F_{ij} in **F**. Uzawa [23] showed that Allen partial elasticities of substitution

Table III. Elasticity of Substitution Estimates from Translog Cost Function

	Police	Civilian	Capital
Police	−0.1149 (.0298)	0.9032 (.0427)	0.2368 (.1693)
Civilian		−6.8894 (5.0371)	−1.9609 (2.4961)
Capital			−0.1591 (1.5454)

Notes: Elasticities calculated at the means of the variables.
Asymptotic standard errors in parentheses.

can be computed from $\sigma_{ij} = CC_{ij}/C_iC_j$, so that for the translog cost function, APES is given as:

$$\begin{aligned} \sigma_{ij}^{A} &= (\beta_{ij} + S_iS_j)/S_iS_j, \qquad i \neq j \\ \sigma_{ii}^{A} &= [(\beta_{ii} + S_i(S_i - 1)]/S_i^2, \qquad i = j \end{aligned} \tag{7}$$

where S_i, S_j are the cost shares of factors i, j in total cost. Since **F** is symmetric, the matrix of partial elasticity of substitution will also be symmetric. These elasticities of substitution are nonlinear functions of the estimated parameters; thus their standard errors cannot be calculated exactly. However, under the assumption that the shares (S_i) are constant and equal to the means of their estimated values, we can obtain approximate estimates of the standard errors [21]:

$$\begin{aligned} \mathrm{var}(\sigma_{ij}^{A}) &= \mathrm{var}(\beta_{ij})/S_i^2S_j^2, \qquad i \neq j \\ \mathrm{var}(\sigma_{ii}^{A}) &= \mathrm{var}(\beta_{ii})/S_i^4, \qquad i = j. \end{aligned} \tag{8}$$

The calculated APES between sworn personnel, civilian employees and capital are shown in Table III. Consider first the own substitution elasticities (the diagonal elements in Table III). They are all negative and significant as expected. The estimated cross substitution elasticities (the off-diagonal elements in Table III) shows a relatively high degree of substitutability between police and civilian. We also find that the elasticity of substitution for police and capital is positive, so that these inputs are substitutes. This result is not surprising since capital inputs such as computers and electronic surveillance devices, do cut down on manpower needs. It shows considerable feasibility of substituting capital for officers on the beat, thus making it possible for the same area to be patrolled by fewer officers.

Civilian employees and capital are found to be complements with a calculated APES of −1.9609. Given that the civilian employees serve in administrative and clerical positions in police departments, the complementarity observed between the two inputs is a reasonable one. Civilian employees have to be provided with the necessary capital. Also civilians are employed to operate certain types of capital such as computers, word processors and typewriters, for which police officers may not have the requisite skills. Hence the increasing use of such capital equipment should boost the employment of civilian employees. These results are similar to those obtained by Gyimah-Brempong [10].

Most of our calculated partial (cross) elasticities of substitution are different from unity, so that the underlying production function is not Cobb-Douglas. Researchers who have employed the Cobb-Douglas functional form are very close to our substitution elasticities only when considering

Table IV. Own and Cross-Price Elasticities of Input Demand

	Police	Civilian	Capital
Police	−0.0917	0.7211	0.1891
	(.0239)	(.0001)	(.0229)
Civilian	0.0596	−0.4548	−0.1295
	(.0343)	(.3326)	(.3384)
Capital	0.0321	−0.2658	−0.0216
	(.1352)	(.1652)	(.2101)

Note: Standard errors in parentheses.

substitution between police and civilian employment. Also the calculated APES differ in important ways from that of Phillips [19]. Our calculated substitution elasticities are smaller in absolute magnitude than his estimates for Long Beach, San Diego and Oakland, but higher than his estimates for Los Angeles. Also, contrary to his findings of substitutability between civilian and capital, we find these inputs to be complements.

The information contained in Table III can be used to test for the existence of a consistent aggregate index of labor inputs. For example, is it possible to collapse police and civilian labor inputs into one aggregate labor input without affecting the estimated coefficients? Berndt and Christensen [3] have shown that a consistent aggregation index for two inputs X_i and X_j exists if the APES of the two inputs with a third input X_k are equal, i.e., $\sigma_{ik}^A = \sigma_{jk}^A$. In our model, a consistent index of aggregation of labor inputs exists if the APES between police and capital equals the APES between civilian and capital. As the results from Table III show, the Berndt-Christensen condition for aggregation does not appear to be satisfied.

Another criteria for checking for the existence of an aggregation index is to apply the Hicks-Leontieff aggregation theorem. The Hicks-Leontieff aggregation condition states that a consistent index of aggregation exists for a group of commodities if their prices change in the same proportion; in other words, that they are perfectly correlated. The simple correlation between police wage and civilian wage is −0.3041, which suggests that for our data W_p and W_{civ} do not change in the same proportion. Our results imply that no consistent aggregate index exists for police and civilian inputs.

Finally, from the computed APES, we calculate own- and cross-price elasticities of input demands. Price elasticity of demand is related to the APES in the following manner:

$$\eta_{ij} = \sigma_{ij}^A S_j \qquad \text{for } i \neq j$$

and

$$\eta_{ii} = \sigma_{ii}^A S_i \qquad \text{for } i = j \tag{9}$$

where S_i is the share of input i in total cost. The asymptotic variances of the price elasticities of demand for inputs is calculated as [21]:

$$\text{var}(\eta_{ij}) = \text{var}(\beta_{ij})/S_i^2 \qquad \text{for } i \neq j$$

and

$$\text{var}(\eta_{ii}) = \text{var}(\beta_{ii})/S_i^2 \qquad \text{for } i = j. \tag{10}$$

The estimates of own-and-cross price elasticities are shown in Table IV together with their asymptotic standard errors. Since they are not necessarily symmetrical we have listed all nine elasticities. The own price elasticity of demand (PEDs), the diagonal elements, are largest for civilians and least for capital. That the PEDs for capital is close to zero suggests that little sensitivity to price changes exists at this level, while there appear to be greater scope for substitution for civilian employees. The PEDs suggests that demand for all three inputs are generally inelastic. This implies that, in spite of the possibilities of substitution among the inputs in production, total expenditures on the provision of law enforcement is likely to increase at a faster pace than the demand for law enforcement.

V. Returns to Scale and Economies of Scope

One aspect of production functions that is of interest to policy makers is returns to scale. For the production of local public services such as police, returns to scale has implications for the organization of local governments. For example, if police production functions exhibit increasing returns to scale, consolidation of police departments in a metropolitan area will decrease the unit cost of police services. We estimate scale economies (SCE) in this section.

In estimating scale economies, we mean-scale the variables.[4] The logarithm of the variables at their means are therefore equal to zero. We use the scaled variables to estimate the full model and derive *SCE* from the parameter estimates. From a cost function, economies of scale is measured as unity minus the proportionate change in total cost when output changes by any quantity [5]. Formally:

$$SCE = 1 - \partial \ln C / \partial \ln Y \tag{11}$$

where C = total cost, and Y = output. The production function exhibits increasing, constant, or decreasing returns to scale depending on whether *SCE* is greater than, equal to, or less than zero.

For a multiproduct firm with joint cost, it does not make sense to define *SCE* for one product. For a multiproduct cost function, therefore, *SCE* is defined as unity minus the proportionate change in cost resulting from a proportionate change in all outputs. SCE is given as:

$$SCE = 1 - \sum_{i} \partial \ln C / \partial \ln Y_i . \tag{12}$$

For the multiproduct translog cost function with 7 outputs and 3 inputs, this implies that SCE is given as:

$$SCE = 1 - \Big(\sum_{i=1}^{7} a_i + \sum_{i}^{7}\sum_{j}^{7} a_{ij} \ln Y_j + \sum_{j}^{7}\sum_{j}^{3} b_{jk} \ln W_k\Big) \tag{13}$$

Because the variables are mean-scaled, the last two expressions in parentheses reduce to zero. This implies that our measure of *SCE* is $1 - \Sigma_{i=1}^{7} a_i$. From the output coefficients, we calculate *SCE* to be 0.0774, indicating increasing returns to scale.[5] The positive value of *SCE* seems

4. We mean scaled the variables to make the calculation of $\check{S}CE$ more tractable.

5. The first-order output coefficients (absolute t statistics in parentheses) from the regression equation are as follows:

Table V. Statistics for Testing for the Existence of EOS

Hypothesis	Degrees of Freedom	Test Statistic
No Economies of Scope*		
i. $\alpha_{12} + \alpha_1\alpha_2 = 0$	1	12.2342
ii. $\alpha_{13} + \alpha_1\alpha_3 = 0$	1	6.0203
iii. $\alpha_{14} + \alpha_1\alpha_4 = 0$	1	3.9589
iv. $\alpha_{15} + \alpha_1\alpha_5 = 0$	1	9.1355
v. $\alpha_{16} + \alpha_1\alpha_6 = 0$	1	5.5132
vi. $\alpha_{17} + \alpha_1\alpha_7 = 0$	1	4.6024
vii. $\alpha_{23} + \alpha_2\alpha_3 = 0$	1	7.9486
viii. $\alpha_{24} + \alpha_2\alpha_4 = 0$	1	3.9985
ix. $\alpha_{25} + \alpha_2\alpha_5 = 0$	1	3.3869
x. $\alpha_{26} + \alpha_2\alpha_6 = 0$	1	5.2635
xi. $\alpha_{27} + \alpha_2\alpha_7 = 0$	1	4.3065
xii. $\alpha_{34} + \alpha_3\alpha_4 = 0$	1	5.5286
xiii. $\alpha_{35} + \alpha_3\alpha_5 = 0$	1	6.1754
xiv. $\alpha_{36} + \alpha_3\alpha_6 = 0$	1	3.5321
xv. $\alpha_{37} + \alpha_3\alpha_7 = 0$	1	4.0073
xvi. $\alpha_{45} + \alpha_4\alpha_5 = 0$	1	5.4670
xvii. $\alpha_{46} + \alpha_4\alpha_6 = 0$	1	3.6795
xviii. $\alpha_{47} + \alpha_4\alpha_7 = 0$	1	3.5266
xix. $\alpha_{56} + \alpha_5\alpha_6 = 0$	1	12.3233
xx. $\alpha_{57} + \alpha_5\alpha_7 = 0$	1	5.1491
xxi. $\alpha_{67} + \alpha_6\alpha_7 = 0$	1	3.7213

*Outputs labelled as follows: rob = 1, burg = 2, larc = 3, mvth = 4, pers = 5, pop = 6, and ars = 7.

to imply that police production in Michigan exhibits increasing returns to scale. The question, however, is whether the calculated SCE is significantly different from zero. From information contained in the variance-covariance matrix, we calculate the standard error of SCE to be 0.1353. This gives us a t statistic of 0.5721. We cannot therefore reject the null hypothesis that SCE is zero at any reasonable level of significance. This result implies that no economies in police production can be gained by consolidating municipal police departments in Michigan.

Though we did not find economies of scale in Michigan municipal police departments, it is possible that there exists economies of scope due to interproduct complementarity. Economies of Scope (EOS) exists when the cost of jointly producing two or more goods is less than the total cost of producing these outputs separately. Formally, EOS exists if:

$$C(Y_i, Y_j) < \left[C(Y_i, 0) + C(0, Y_j)\right]. \tag{14}$$

A multiproduct cost function exhibits EOS if

$$\partial^2/\partial Y_i \partial Y_j < 0$$

$$a_1 = 0.1157(1.20),\quad a_2 = -0.3016(1.93),\quad a_3 = -0.1807(2.32),$$
$$a_4 = 0.0787(0.75),\quad a_5 = 0.1956(1.85),\quad a_6 = 0.8973(7.05),$$
$$a_7 = 0.1176(1.56).$$

Parameter estimates for the full model is available from the authors upon request.

for $i \neq j$; $i,j = 1, \ldots n$. For the multiproduct translog cost function, a test for this condition is [18]:

$$\alpha_i \alpha_j + \alpha_{ij} < 0$$

where the variables are normalized around their means.

Table V shows the likelihood ratio test for the null hypothesis that there exists no EOS among the 7 police outputs. At $\alpha = .01$, there exists EOS between rob and burg, rob and pers, burg and larc, and pers and pop. However, if we test the null hypothesis of no EOS at $\alpha = .05$, then the null hypothesis cannot be rejected in only 5 cases—burg and pers, larc and pop, mvt and pop, mvt and ars, and pop and pers. This means that there is considerable amount of interproduct complementarity in police production. Therefore joint production of these outputs decreases the cost of production.

VI. Conclusion

This paper investigated the structure of police production function in Michigan municipal police departments. Using a multiproduct translog cost function, we find that the underlying production function is not of the Cobb-Douglas functional form and that the elasticity of factor substitution is significantly different from unity. We also find that civilian labor and capital inputs are complementary. These results imply that researchers who have used the Cobb-Douglas functional form may have misspecified their models.

We find no consistent index of labor aggregation in police production and no evidence of statistically significant scale economies. Thus no economies will be reaped by consolidation of police production in metropolitan areas. We, however, found the existence of economies of scope, implying that joint production of police outputs reduces production cost.

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