A Simulation Model of the Effect of Fertility Reduction on Economic Growth in Africa

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Abstract

We investigate the effects of a decline in fertility on economic growth and development outcomes using a macrosimulation model. We incorporate three fertility effects that have previously not been included in such models: the effect of fertility on child health and later worker productivity; the effect of fertility on savings; and a feedback mechanism from female education to fertility, in which changes in female education that are induced by declining fertility in turn alter subsequent fertility. We also improve the model of the economy by incorporating a more realistic three-sector framework and by allowing for labor market imperfections. Using data from Nigeria, we find that adding these channels roughly doubles the effect of an initial fertility decline on income per capita after 50 years when compared to previous simulation results.

1. *Introduction*

The demographic transition from high mortality and high fertility to low mortality and low fertility is well underway around the world and has started in Sub-Saharan Africa in recent decades (Angeles, 2010; Bloom, Canning, & Sevilla, 2003; Caldwell, Orubuloye, & Caldwell, 1992). There is evidence that the decline in fertility, which accompanies the latter stages of the demographic transition, creates the potential for a "demographic dividend" and a window of opportunity for economic growth (Bloom, Canning, Fink, & Finlay, 2007, 2010; Bloom et al., 2003; Sinding, 2009). In addition to the mechanical increase in income from the decline in youth dependency rates and rise in working age share of the population, the decline in fertility leads to changes in behavior that can lead to higher income. One important mechanism is that lower fertility can induce higher labor force participation rates, particularly for women (Bloom, Canning, Fink, & Finlay, 2009). Reduced youth dependency rates may also lead to increased investment in the health and education of each child, thereby increasing their productivity when they enter the workforce (Becker, Duesenberry, & Okun, 1960; Becker & Lewis, 1973; Bloom, Canning, Fink, et al., 2010). Changes in fertility and age structure may also affect national savings rates and investment (Deaton & Paxson, 1997; Kelley & Schmidt, 1996; Lee, Mason, & Miller, 2001). Finally, there may also be a positive feedback effect between the demographic and economic transitions, when fertility decline induces improvements in health, education, female labor market participation, and economic growth, and when these improvements in turn lead to further reductions in fertility and additional economic benefits.

While cross-country regression models suggest positive effects of fertility and age structure on economic growth (Barro, 1991; Bloom & Canning, 2008; Mankiw, Romer, & Weil, 1992), these aggregate models do not usually identify the channels through which fertility works, and their applicability to particular countries may be problematic since they lack the ability to model country specific factors in detail. An alternative approach, which we follow in this study, is to construct a macrosimulation model of economic growth and to parameterize the mechanisms in the model from microeconomic studies along the lines used by Moreland, Madson, Kuang, Hamilton, & Jurczynska (2014) and Ashraf, Weil, & Wilde (2013). Our approach is based on the work of Ashraf, Weil, & Wilde (2013), who examine the economic effects of fertility decline through changes in age structure, female labor force participation, investment in children's education, and increases in the capital-labor ratio.

In our analysis and modelling, we add three key mechanisms that have not been previously considered in this body of work.

Firstly, we add a channel that links fertility decline to improved health outcomes for children. Through this channel, smaller family sizes and increased intervals between births may allow for additional health investments in children which, in turn, can contribute to physical and cognitive development and can lead to increases in human capital and improved worker productivity (Canning & Schultz, 2012; Cleland, Conde-Agudelo, Peterson, Ross, & Tsui, 2012).

Secondly, we incorporate a mechanism through which the change in the population age structure due to fertility decline may increase savings rates. At the household level savings rates vary with age, with a peak during people's working lives, so that at the national level aggregate savings will depend on the age structure of the population (Bloom, Canning, Mansfield, & Moore, 2007; Higgins, 1998; Lee et al., 2001; Leff, 1969). There may also be an additional effect of lower fertility on expected transfers from children to their elderly parents, increasing the need for savings for retirement (Smith & Orcutt, 1980; Weil, 1994). Higher savings rates from reductions in fertility rates may, in turn, boost the capital-labor ratio over and above the effect of having smaller inflows of working age people[1].

In our model, we consider the effect of an initial decline in fertility brought about by an increase in contraceptive use through improved access to family planning. However, we also add the possibility of subsequent further fertility reductions as fertility reacts endogenously to induced changes in economic conditions. In particular, if fertility decline leads to an increase in educational investments in children, these higher levels of education can reduce fertility in the next generation. This important feedback channel implies that the effects of the initial decline in fertility are compounded by further reductions due to rising levels of female education (Drèze & Murthi, 2001). However this feedback mechanisms is slow in coming, as it only occurs when the child cohorts with increased educational attainment reach childbearing age (Cleland & Rodríguez, 1988; Diamond, Newby, & Varle, 1999; Osili & Long, 2008).

[1] Further details on the fertility-savings relationship in our model are described in later sections and in Appendix 2.

In addition to adding these three additional mechanisms, we also develop the economic structure of the model to make it more realistic. Previous simulation approaches, including the model by Ashraf et al. (2013), assume a one-sector model of the economy and perfect markets so there is full employment. In such a model the supply side effects of demographic change on labor and capital are automatically turned into increased output. However, evidence from cross-country studies shows that the demographic dividend is not automatic but rather depends on their being appropriate economic policies in place to produce adequate demand for the resources produced by the supply side (Bloom, Canning, Fink, et al., 2007). One way of allowing for market imperfections would be to allow for unemployment in the model so that so that increases in labor supply may lead to mass unemployment rather than higher output.

While modeling mass unemployment in response to a rapidly increasing labor supply may be appropriate for developed countries, it does not appear to be appropriate for poor developing countries in Sub-Saharan Africa. In most of Sub-Saharan Africa, the lack of unemployment insurance implies that people are compelled to work even if the wage that they earn is low (Bigsten & Horton, 2009; Goldin, 1994). In this setting, the effect of rapid population growth may be to drive more workers into low wage and low productivity jobs in labor-intensive traditional sectors of the labor market, particularly in agriculture, rather than to create unemployment. To model this, we use the Lewis (1954) model of developing economies and assume the economy is comprised of three sectors. The first of these sectors we take to be the "modern" part of the economy that encompasses industrial sectors such as manufacturing, sectors that demand skilled labor, and the formal service industries. In this sector, physical capital and labor augmented by human capital (in the form of education and health) are used in for production, and workers are paid wages that are equal to their marginal product. The second sector, which we refer to as the "traditional" sector, represents the labor-intensive part of the economy that uses labor and land as input factors of production. This traditional sector consists mainly of subsistence agriculture and low skilled services such as roadside trading, though some agriculture and services are either physical or human capital intensive and should be thought of as being in the modern sector. Like prior single-sector models, we also include a fixed factor, land, which can generate Malthusian crowding effects if there is rapid population growth; however, this effect only occurs in the traditional sector in our model. In addition, we do not assume that the wages equalize across sectors. Rather, wages are higher in the modern sector than in the traditional sector, and we impose a fixed wedge between the earnings in each sector, which reflects the cost of migration and other distortions such as taxes that are levied on the modern sector but not on the traditional sector. The equilibrium in the model is inefficient due to worker productivity and real wages being higher in the modern sector than in the traditional sector, which reflects a standard stylized fact that is observed in developing countries (Bloom, Canning, Hu, et al., 2010). Finally, we also allow for an exogenous contribution of a raw material sector to output, which is often an important contribution to national income in many Sub-Saharan African countries. These changes, when taken together, allow our model to more realistically reflect Sub-Saharan African economies than a single sector model with complete efficiency.

In our simulation analysis, we begin with a "baseline" scenario in which the time path of fertility follows the United Nations Population Projections (UNPP) high fertility variant forecast (United Nations, 2013). We then compare the outcomes under the introduction of an intensive family planning program that lowers fertility. We assume that the intensity of the program is sufficient to bring fertility down to the UN low fertility variant forecast, in which the total fertility rate falls by 0.5 births per woman after 5 years, 0.8 births per woman after 10 years, and one birth after 15 years and thereafter from the start of the projection period. This reduction in fertility is consistent with estimates of the effect of family planning programs in Matlab, Bangladesh in the 1980s and in Navrongo, Ghana, in the 1990s (Debpuur et al., 2002; Joshi & Schultz, 2007; Phillips et al., 2012), where changes in fertility in treatment areas were compared to changes in fertility in control areas that did not receive the family planning intervention, and the effect on total fertility rate appeared to have been a reduction of about one child per woman. We feed data from these two fertility scenarios into our model framework, and we run our simulation model to observe the differences in outcomes under each fertility scenario through each of the demographic and economic mechanisms outlined above, including feedbacks into further induced fertility decline.

2. The Model

We now outline the structure of the model. Additional details of the model, including the precise equations we use, are given in Appendix 1. We consider a model of the demographic dividend in Sub-Saharan Africa, which gives rise to some issues that might not be present in developed countries. In particular, we allow for a three sector model with a highly productive modern sector that uses physical capital, human capital, and labor, a traditional sector that uses land and labor, and a raw material sector that requires no inputs.

Population and Effective Labor

The base of our model is similar to that of Ashraf et al. (2013). We define each period in our model to be five years, and we divide population into five-year age groups. We calibrate age specific mortality rates to be consistent with the evolution of age groups from the United Nations World Population Prospects 2012 (United Nations, 2013). These age group specific mortality rates in each future five-year period decline over time in each country but are assumed to be the same across each scenario. For fertility, we begin with the UNPP high fertility scenario as our baseline. We then consider a family planning intervention that reduces fertility gradually over time. The total fertility rate between the two scenarios initially differs by 0.5 births per woman after 5 years from the start of the projection period, then by 0.8 births per woman after 10 years from the start of the projection period, and then finally by one birth after 15 years from the start of the projection period, continuing to differ by one birth between the two scenarios until the end of the projection period. When calculating the population distribution by sex under each of our scenarios, we adhere to the UN projections of the sex ratio at birth within each age group and over time.

Figure 1: Modeling Fertility, Population by Age, and Labor Supply

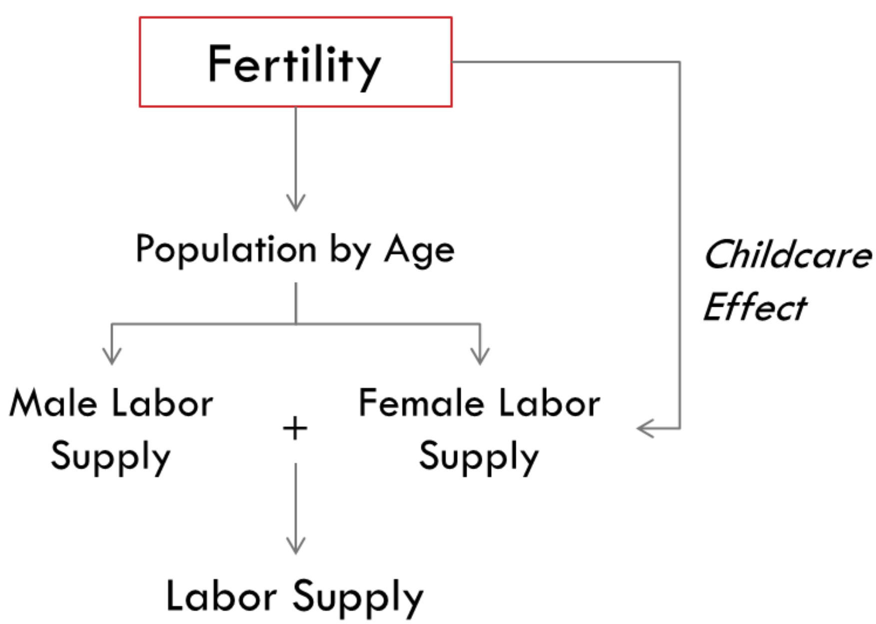
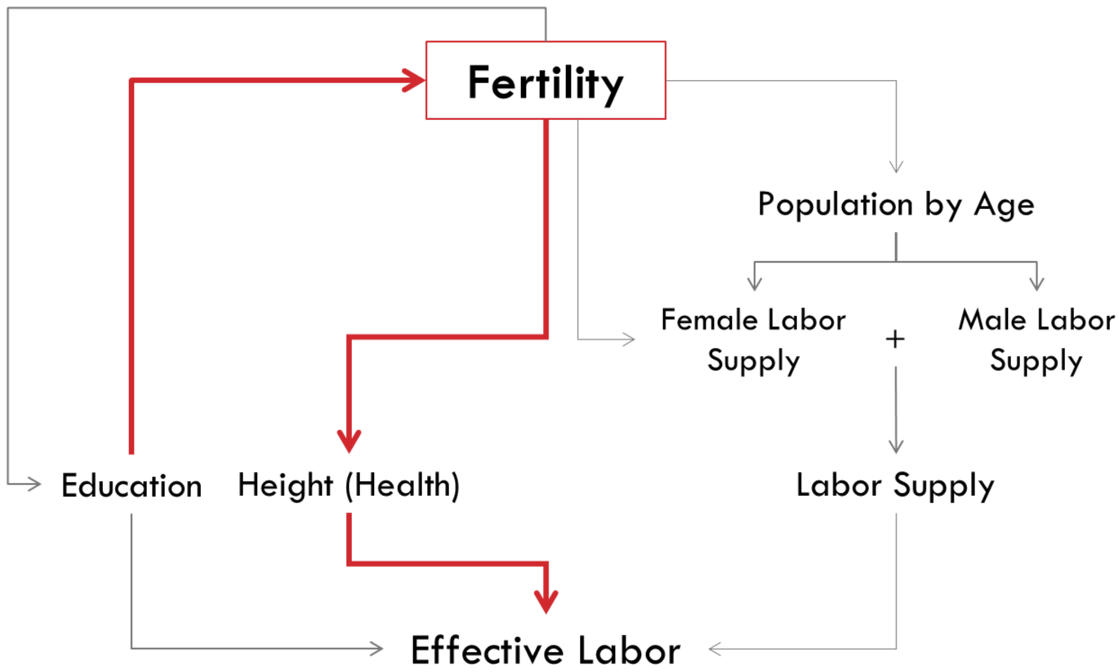


Figure 1 illustrates the main demographic model and shows how we feed our fertility and mortality projections into a population model to obtain estimates of the population by five year age group and by sex in each period under each fertility scenario. We calculate the labor force by assuming that adults enter the labor force at age 20 and leave the labor force at 65. The labor supply contribution by sex in each period is the size of the projected sex-specific population in that age group weighted by the sex and age group specific labor force participation rates in that period. Labor force participation rates are obtained from the International Labour Office's ILOSTAT database (International Labour Office (ILO), 2013) for the year 2010. We assume age specific male participation rates are fixed at this level over time, but we modify the age-specific female labor force participation rate in each period to reflect the impact of fertility change and women's substitution between childcare and work on total female labor supply.

We then model the effects of fertility and demographic change on human capital accumulation, which we capture through effects on both child health as well as education. We assume that a given sex specific cohort's educational attainment and health stock (quantified in average years of schooling per individual and adult height, respectively) are entirely amassed before age 20, after which the average level of schooling and average adult height for that cohort are held constant for the remainder of that cohort's lifetime. We then parametrize the fertility-to-education and fertility-to-health relationships to capture the "quality-quantity" frontier in which investment per child in education and health rises as the number of children falls (Becker, 1981; Becker & Lewis, 1973; Lam, 2003).

In contrast to previous macrosimulation modeling approaches, we endogenize the evolution of fertility over time through a feedback channel from female education to fertility. This feedback channel further reduces the fertility rate in the low fertility scenario relative to the high fertility scenario as increased female education feeds into lower fertility. In our model, we calculate average years of schooling and average height in each period separately for each sex as weighted sums of the average years of schooling and average height of each cohort, respectively. We then combine the sex-specific estimates in a weighted average to estimate the level human capital that is accumulated for the entire workforce for that period, and we then combine these human capital estimates with our projection of the size labor force to predict effective labor over time. Figure 2 outlines the process for deriving effective labor in our model and highlights the new channels (endogenous education feedback, health) in red.

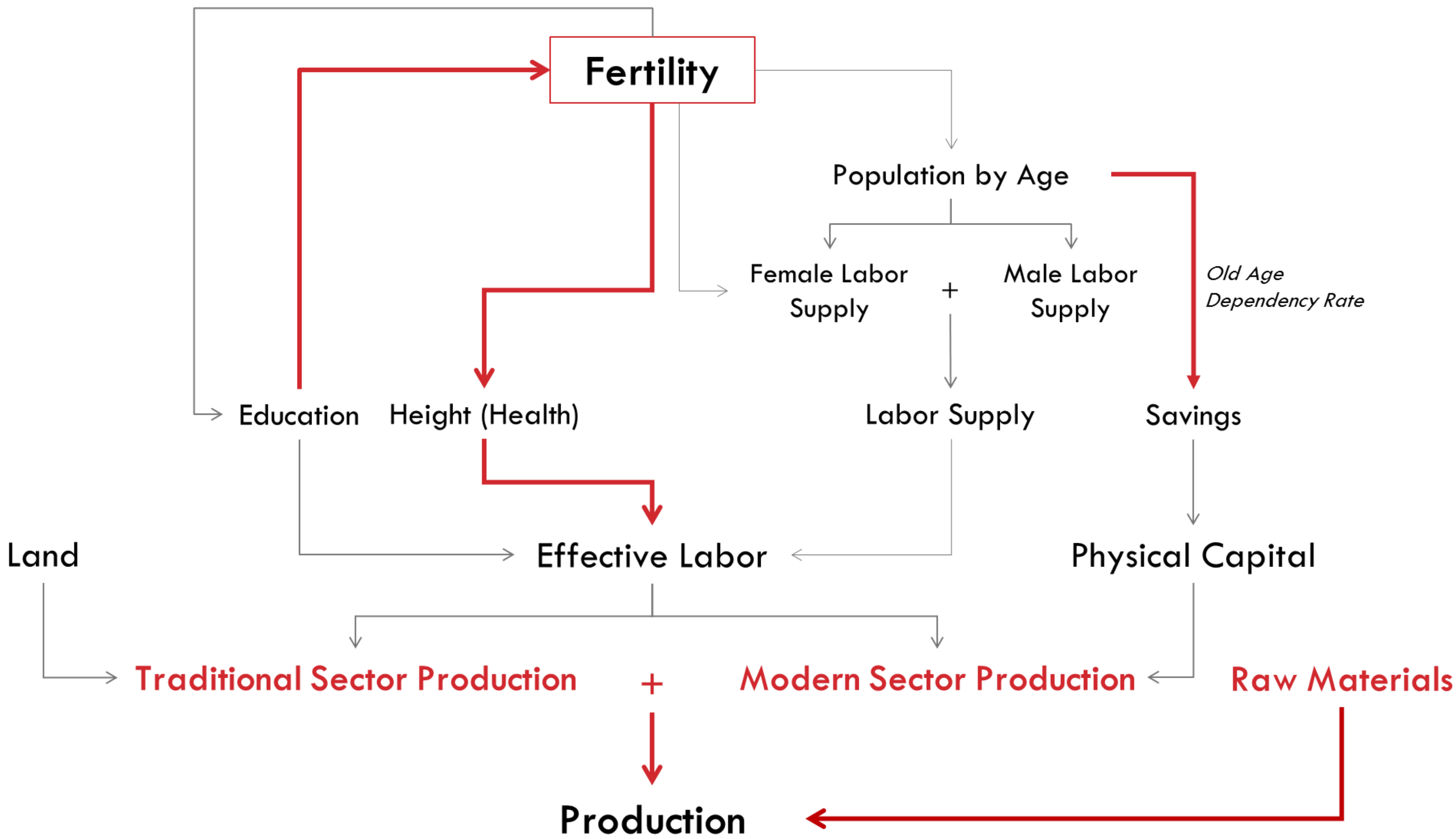
Figure 2: Effective Labor



Production

Figure 3 presents our full demographic-economic model of production. Estimates of education, health, and labor supply from our demographic simulations, which together comprise effective labor, are fed into our model along with capital and land. We extend beyond previous one sector model approaches by considering a Lewis development economy with three sectors, a modern sector, a "traditional" sector, and a raw materials sector. Labor is divided between the two sectors. Production in the modern sector is given by a standard Cobb-Douglas production function, with inputs of physical capital, labor allocated to the modern sector, and human capital in the form of average years of schooling in the workforce (as a proxy for education), and average height of the workforce (as a proxy for health). In a similar fashion, aggregate production in the traditional sector is also modeled by a Cobb-Douglas production function, with agricultural land and labor allocated to the traditional sector as factor inputs. The stock of agricultural land in our model is assumed to be fixed, but we acknowledge that there may be variable returns to land through advances in agriculture technologies (e.g. improved farming techniques, more productive methods of irrigation, etc.), land reclamation and improvement, and through more effective use of natural resources. In addition, it is also possible that the extensive margin of land cultivation may change as a result of population pressure; however, to estimate the variable returns to land and the substitutability between land and other factors of production, particularly across different countries and over time, is difficult. We assume that the traditional sector does not use physical or human capital and model it to capture subsistence agricultural and low skill production in the informal sectors. Finally, we allow for a raw material sector (e.g. oil or mineral production) that produces output exogenously of other inputs. While this sector does require both capital and labor inputs, in practice, it is not very labor intensive, and income from this sector comes almost entirely from a country's endowment of natural capital (Ross, 2012). We therefore abstract away from modeling output in this sector and include production from raw materials as a constant additive term in total output.

Figure 3: Full Demographic-Economic Model of Production



Capital Accumulation and Savings

We replicate a Solow framework for capital accumulation, assuming that net investment depends on the existing aggregate output weighted by the savings rate and net of the depreciation of the existing level of capital stock. Following Bloom, Canning, Mansfield, et al. (2007), we model the evolution of the savings rate is a function of the past savings rate, the level of income, and age structure in the form of the ratio of old-age dependents to the working age population. By modeling savings in this manner, we capture the idea that savings behavior depends on age, where peak savings occurs when people are prime age workers and declines with age to the point where the old dis-save. The level of income has an important impact on savings; in very poor countries, there is little life cycle saving, and retirement and saving for retirement are luxury goods and behaviors that only emerge once income levels are sufficiently high. Bloom, Canning, Mansfield, et al. (2007) also emphasize that savings are dependent on incentives from social security systems; pay-as-you-go pension systems can generate income for retirement without the need for real savings to be accumulated. We assume that that such pension systems are not operational in our model.

Labor Allocation across Sectors

Our model specification requires that modern sector and traditional sector wages, which endogenously adjust within their respective labor markets, will in turn determine equilibrium labor supply allocations across the two sectors that employ workers. The wage rate[2] in the modern sector in a given period is set to be equal to the marginal product of labor in the modern sector for an additional worker with average levels of education and health. However, in following Lewis (1954)'s dual-sector model of surplus labor, we assume that the traditional sector is not based on a market mechanisms but involves sharing of output among family members. Hence, the wage per worker in the traditional sector will be determined by the average product of that sector. This wage condition captures a common observation in low-income countries in which family members share incomes and communities pool and divide resources as a means of insuring against risk (Cypher & Dietz, 2009; Lewis, 1954).

We assume that labor moves between sectors so that the net effective earnings are equalized across sectors. Since the wage in the traditional sector is determined at the average and not at the margin, in equilibrium the marginal productivity in the traditional sector will be lower than in the modern sector.

[2] Throughout this paper, we use the term "wage" to describe the wage rate per worker. This is distinct from the total wage bill, which can be calculated by multiplying the wage rate by the total number of workers.

In addition, there may be costs, such as migration costs, labor and employer taxes, or bribes to corrupt officials, which are levied on workers or employers in the modern sector but not on workers in the traditional sector and which, in turn, will discourage traditional sector workers from entering the modern sector. These costs will also contribute to an inefficient allocation of labor across sectors. We assume workers will migrate between sectors to establish an equilibrium where wages in the modern sector, net of all costs, are equal to the wage in the traditional sector.

3. Calibration

Table 1 describes each parameter that is used in the model, the parameter values that were used to calibrate the model, and the sources from which these values were obtained.

Table 1: Parameter Calibration

Parameter Symbol	Value	Description	Source(s)
π	0.02	Effect of fertility on female labor supply	Ashraf et al. (2013)
θ_E	0.2	Effect of fertility on childhood education	Joshi & Schultz (2007); Rosenzweig & Wolpin (1980)
ψ	-0.15	Effect of women's education on fertility	Osili & Long (2008)
θ_H	-0.00067	Effect of fertility on adult height	Giroux (2008); Joshi & Schultz (2013); Kravdal & Kodzi (2011); Stevens et al. (2012); Victora et al. (2008)
α	0.33	Capital share of output in modern sector	Hall & Jones (1999)
β	0.167	Land share of output in traditional sector	Kawagoe et al. (1985); Williamson (1998, 2002)
γ	0.1	Economic returns to schooling	Banerjee & Duflo (2005); Oyelere (2010); Psacharopoulos (1994); Psacharopoulos & Patrinos (2004)
λ	0.08	Effect of health on output	Schultz (2002, 2005)
δ	0.07	Depreciation rate of capital	Schmitt-Grohe & Uribe (2006)
ϕ_1	0.758	Effect of lagged savings on current savings	Bloom et al. (2007)
ϕ_2	0.133	Effect of wage rate on savings rate	Bloom et al. (2007)
ϕ_3	-0.006	Effect of squared wage rate on savings rate	Bloom et al. (2007)
ϕ_4	-0.209	Effect of ratio of old to working age population on savings rate	Bloom et al. (2007)

Estimates of key parameters that illustrate the direct relationships between fertility and other factors are drawn from several sources. To identify the direct time cost and reduction in labor market participation due to an additional child, π, we follow the parameterization approach described in Ashraf et al. (2013), who interpolate Filipino data from Tiefenthaler (1997) and find that lifetime female labor supply declines by an estimated 2 percent for each additional birth. This effect is fairly small and is consistent with the fact that female labor market participation in Africa is currently very high and has little scope for increase. In the traditional sector, work is often in the home and can be combined with childcare (Goldin, 1994; Verick, 2014; Westeneng & D'Exelle, 2011), and it may only be in the modern sector that there is a sharp division between home and work and a tradeoff between working and looking after children.

Parameter estimates for the direct effect of fertility on educational attainment, θ_E, are obtained from Rosenzweig & Wolpin (1980) and Joshi & Schultz (2007), who draw upon quasi-experimental evidence from a family planning intervention in Matlab, Bangladesh and find that a 15 percent reduction in total fertility, which is equivalent to having one fewer birth, increases the number of years of schooling in children by 20 percent. When considering the endogenous response of fertility to changes in education, we parameterize the coefficient ψ, the direct effect of education on fertility, using results from Osili & Long (2008), who examined the causal impact of a universal primary education program in Nigeria and found that each additional year of female schooling reduced fertility by 0.26 to 0.48 births, which constitutes a 11 to 19 percent reduction. We obtain our parameter value of 15 percent for ψ by averaging across the various Osili-Long estimates.

We expect that a reduction in fertility will increase the health and nutrition resources available per child and thus lead to improved child health outcomes. There is also evidence from the Matlab family planning experiment to suggest that providing improved access to family planning and child health services both reduced fertility and child mortality (Joshi & Schultz, 2013); however, direct evidence on the effect of fertility on surviving children's health and subsequent worker productivity is limited. We therefore take an indirect approach to calibrate our estimate for θ_H, which captures the impact of fertility on child health and health human capital (as proxied by adult height), by first examining the effect of fertility on child height and stunting, then inferring this effect of child stunting on adult height, and finally estimating the effect of adult height on worker productivity and wages.

Giroux (2008) and Kravdal & Kodzi (2011) examine the effect of fertility and the number of siblings on child stunting Sub-Saharan Africa. While they find a strong association at the aggregate level, their estimates of the effect size are quite small at the household level. Kravdal & Kodzi (2011) use household level data from 23 countries and find that an extra sibling increases the odds of stunting by about 2 percent, while Giroux (2008) provides estimates for 6 countries and finds that the odds of stunting increase by about 3 percent with each additional child. We use the 2 percent estimate for our calibration; however, Kravdal & Kodzi (2011) do find big effects of short birth intervals on the risk of stunting, so there is scope for larger effects of fertility on child height if reductions in fertility lead to both increases in birth intervals as well as reduction in the number of siblings.

Victora et al. (2008) pool results for a set of longitudinal studies to estimate that each reduction of one standard deviation in a young child's height for age score reduces adult height by 3.23 centimeters. Over the last 30 years, the distribution of child height for age has improved and the prevalence of stunting has declined. Stevens et al. (2012) examine trends in the distribution of height for age scores and find that in developing countries over the last 30 years, the average child's height for age score has improved from -1.86 to -1.16 while the prevalence of child stunting (equivalent to a height for age score less than -2) has fallen from 47 percent to 30 percent. Combining these estimates suggests that a reduction in fertility, and sibling numbers, by one birth would increase the average height of adults by around 0.10 centimeters. If we assume an average adult height of 150 centimeters, this one birth reduction effect size would translate into an increase in adult height by about 0.067 percent (0.1 cm ÷ 150 cm × 100 percent).

Standard estimated values for production factor shares are extracted from the economic growth literature, including the capital share of output In the modern sector estimate of $\alpha = \frac{1}{3} = 0.33$ (Hall & Jones, 1999), the land share of output[3] in agriculture estimate of $\beta = \frac{1}{6} = 0.167$ (Kawagoe, Hayami, & Ruttan, 1985; Williamson, 1998, 2002). For the productivity of human capital we use estimates of the effect of schooling on height (measured in years and centimeters respectively) on log wages. We take the education parameter to be $\gamma = 0.1$, which is an approximate average of the estimated returns to schooling (Banerjee & Duflo, 2005; Oyelere, 2010; Psacharopoulos, 1994; Psacharopoulos & Patrinos, 2004), and the health parameter to be $\lambda = 0.08$, which is based on the estimated wage returns to adult height (Schultz, 2002, 2005). In modeling traditional sector output as a function of land and labor, we recognize that our production function for the traditional sector is a simplification

[3] In our parameterization of the land factor share, β, we refer to Kawagoe et al. (1985)'s examination of the agricultural production function, in which the authors estimate an agricultural factor share between 0.1 and 0.2. Given that the parameter is small relative to the factor share in the modern sector, we set β to be 0.167, which yields a simple tractable solution for the allocation of modern sector labor across sectors.LM_t.

of the Kawagoe model since we do not consider the significant contributions of other reproducible factors to output, including livestock, fertilizer, and machinery.

4. Data Sources

Our simulation analysis is focused on considering interventions that alter the path of fertility from what would otherwise occur along a given baseline. We start with the current population age structure in the baseline scenario, and we assume that fertility and mortality will follow the evolution of the United Nations Population Projections baseline high fertility variant forecast. Our model may be easily tailored to consider different baseline and alternative scenarios across different country contexts; for this study, however, we examine baseline and alternative scenarios constructed using demographic data from Nigeria. This approach allows us to better understand the timing by which different demographic-economic channels operate. Our baseline (high-variant) and alternative (low-variant) scenarios are constructed using current United Nations Population Projections data and vital rates from Nigeria, although it is easy to adapt the model by feeding in data from other countries. Baseline data on age-specific fertility rates and projected populations are gathered from 2010 United Nations World Population Prospects estimates (United Nations, 2010).

For our economic model, we collect baseline data for modern sector and traditional sector outputs, modern sector and traditional sector labor inputs, and available land from World Development Indicators estimates (World Bank, 2012), and we use capital stock estimates from the Penn World Tables (Feenstra, Inklaar, & Timmer, 2015). Baseline data on average schooling and average height by sex and age group are obtained from the 2008 Nigeria Demographic and Health Survey (National Population Commission (NPC) [Nigeria] & ICF Macro, 2009), while estimates of age-specific savings rates are gathered from Bloom, Canning, Mansfield, & Moore, (2007). Baseline labor force participation rates are obtained from the International Labour Organization (ILO) ILOSTAT repository (International Labour Office (ILO), 2013).

Table 2 describes each source of data that was used to fore the baseline data for Nigeria.

Table 2: Data Sources

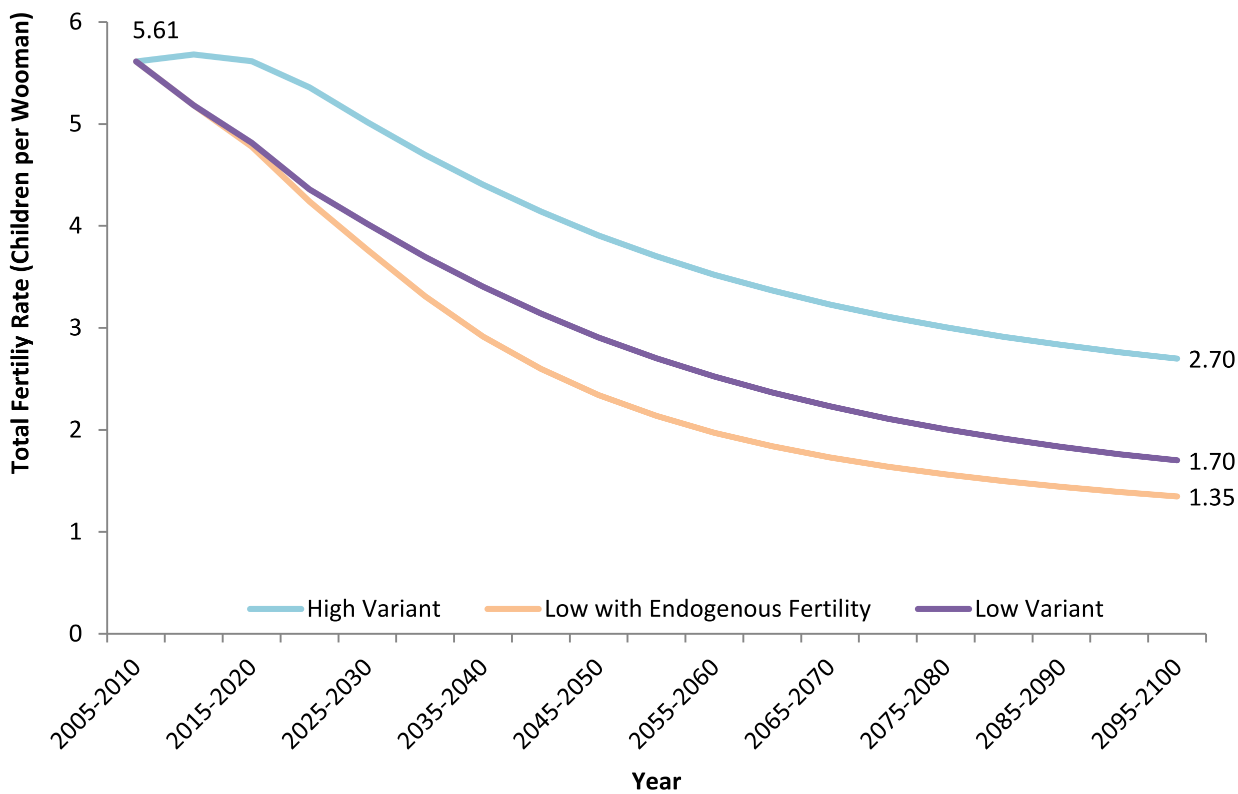
Data Type	Source(s)
Baseline population by age and sex, 2010	UN World Population Prospects (United Nations, 2010)
Baseline age-specific fertility rates, 2010-2100	UN World Population Prospects (United Nations, 2010)
Years of education by 5 year age-sex groups, 2010	2008 Nigeria DHS (National Population Commission (NPC) [Nigeria] & ICF Macro, 2009)
Adult height by 5 year age-sex groups, 2010	2008 Nigeria DHS (National Population Commission (NPC) [Nigeria] & ICF Macro, 2009)
Labor force participation by 5 year age-sex groups, 2010	ILO (International Labour Office (ILO), 2013)
Output, 2005	Penn World Tables (Feenstra et al., 2015)
Output, 2010	Penn World Tables (Feenstra et al., 2015)
Oil Output, 2010	Penn World Tables (Feenstra et al., 2015)
Capital stock, 2010	Penn World Tables (Feenstra et al., 2015)
Agricultural land, 2010	WDI (World Bank, 2012)
Proportion of GDP between modern and traditional sectors, 2010	WDI (World Bank, 2012)
Proportion of labor between modern and traditional sectors, 2010	WDI (World Bank, 2012)

5. *Simulation Results: The Case of Nigeria*

5.1 Demographic Scenario

Figure 4 presents the changing pathways of fertility under the two fertility scenarios. Under the baseline high-variant scenario (the blue line), total fertility declines from an initial 5.61 children per woman in 2005-2010 to 2.70 children per woman by 2095-2100. The total fertility rate under the low-variant scenario (the purple line) progresses on a faster trajectory than the high variant such that the fertility rates between these two scenarios differ by 0.5 births per woman in 2010-2015, 0.80 births per woman in 2015-2020, and by a fixed 1 birth per woman from 2020 onwards.

Figure 4: Total fertility rate (number of children per woman) under baseline high-fertility variant and alternative low-fertility variant scenarios, Nigeria



When accounting for the endogenous responses of fertility from the education channel, we see that the alternative low-variant projection diverges further away from its respective projections that do not incorporate the feedback channel from education to fertility. This divergence is due to the fact that the endogenous feedback from education to fertility is calculated using the high fertility scenario as the reference; feedback effects of education under the low fertility variant scenario are therefore calculated as the additional effect of education on fertility due to deviations in scenario-specific fertility from the high variant. When adjusting for these effects, we see that fertility under the endogenous low-variant scenario is projected to fall by an additional 0.55 births after 50 years and by 0.35 births by the end of the 90-year time horizon. This new pathway is indicated by the orange line in Figure 4.

Figure 5: Population under high-fertility and endogenous low-fertility variant scenarios, Nigeria

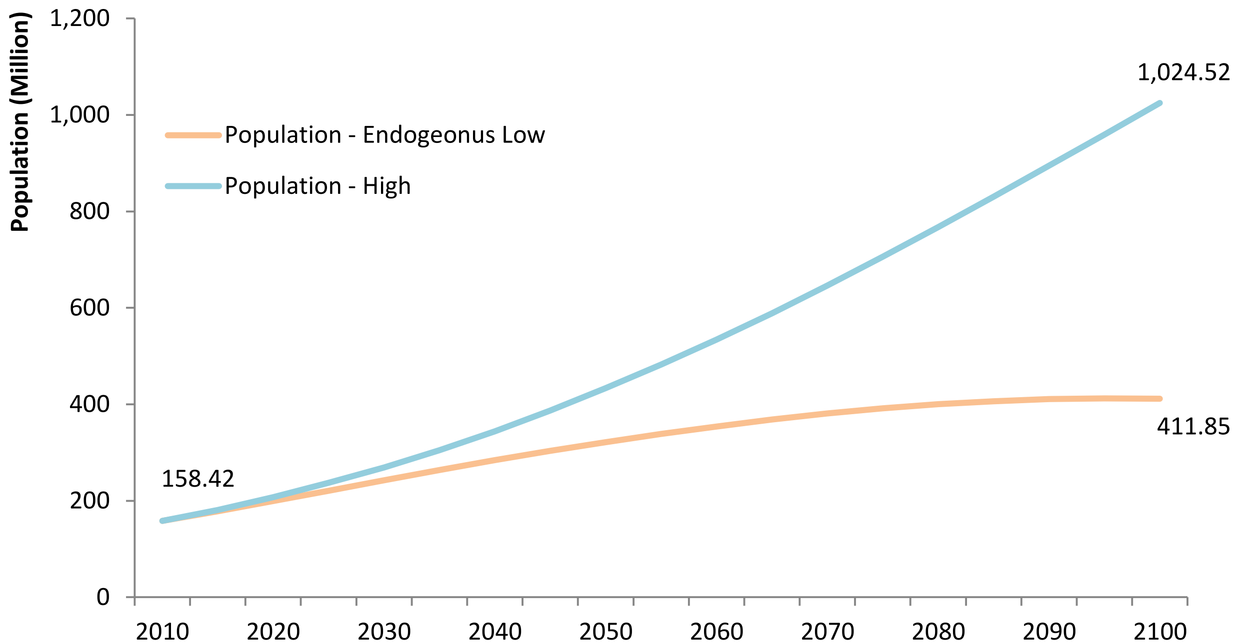
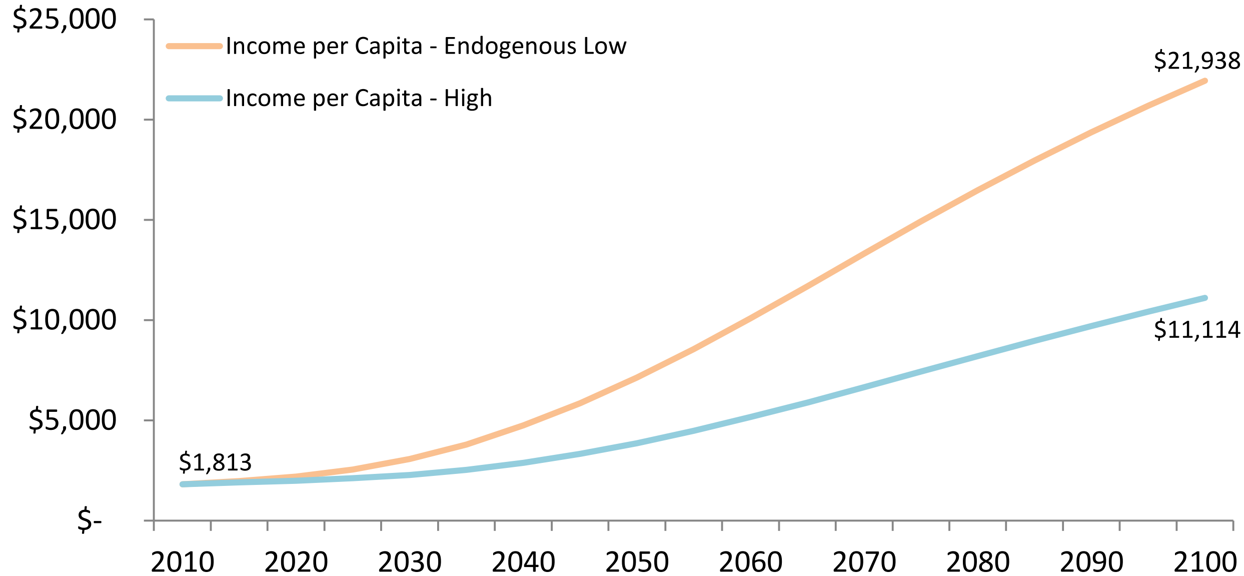


Figure 5 presents the evolution of total population under each of the fertility scenarios. By these estimates, population under the endogenous low-variant scenario will be 25.6 percent lower than the population in the high-variant scenario in 2050 and 59.8 percent lower than the population in the high-variant scenario in 2100.

Figure 6: Per-capita income under high-fertility and endogenous low-fertility variant scenarios, Nigeria



Three-Sector Economic Model Results

Figures 6, 7, and 8 present the path of income per capita, the share of workers in the modern sector as a percentage of total labor supply, and the evolution of modern sector capital per worker (the capital-labor ratio), respectively. Each of these paths is presented under the two fertility scenarios. We refer to the year 2010, which is the last year before total fertility rates in the two scenarios start to diverge, as the starting year for our simulation.

Figure 6 indicates that the reduction of fertility from the high-variant to the endogenous low-variant level of fertility results in an increase in the per-capita income by almost two times (97.4 percent) over a 90-year time horizon. Additionally, we can assume that per-capita income across the two scenarios will continue to diverge because fertility rates in the endogenous low-variant scenario are consistently lower than in the high-variant scenario over the entire period.

Figure 7: Percentage of workers in the modern sector under high-fertility and endogenous low-fertility variant scenarios, Nigeria

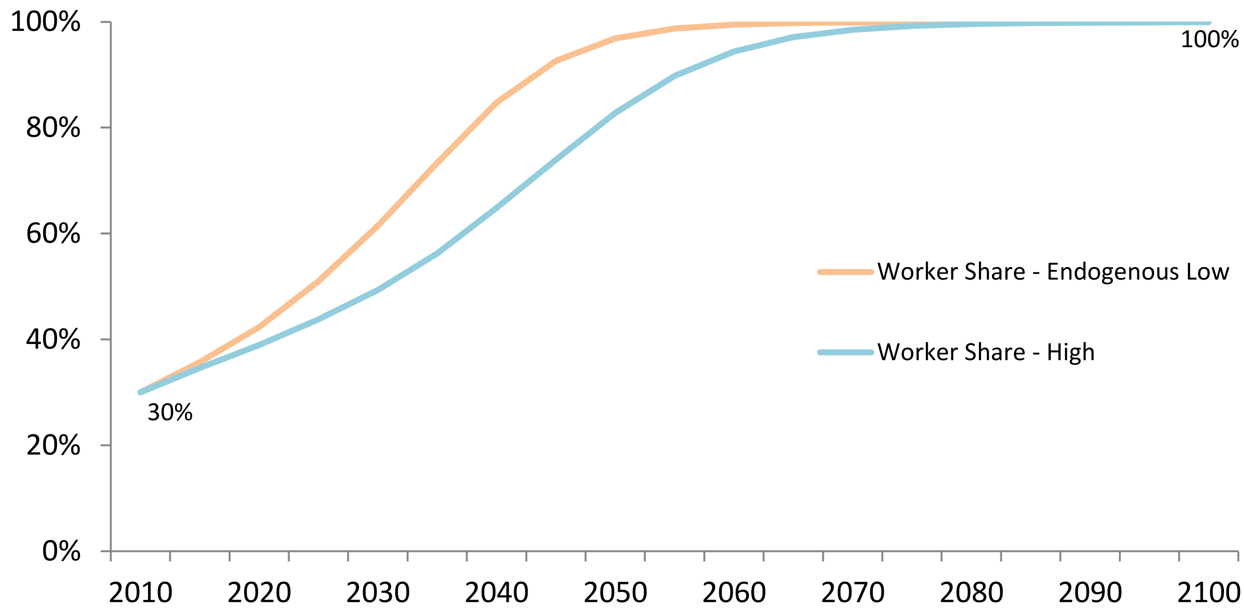


Figure 7 further illustrates the increase in the share of workers in the modern sector as a percentage of the total labor supply. Across both fertility scenarios, we note that share of workers starts out to be smaller in the modern sector than in the traditional sector at only 30 percent of the total labor force. However, beginning around 2025, the share of workers in the modern sector begins to exceed the share of workers in the traditional sector, reflecting the consequent shift in labor and increasing industrialization over time. While both fertility scenarios illustrate this labor transition away from the traditional sector and into the modern sector, the rate at which this labor transition occurs varies considerably by fertility scenario. In particular, the share of workers in the modern sector increases faster and remains higher in the alternative endogenous low-variant fertility scenario compared to the baseline high-variant fertility scenario over the time horizon.

Figure 8: Modern sector capital per worker under high-fertility and endogenous low-fertility variant scenarios, Nigeria

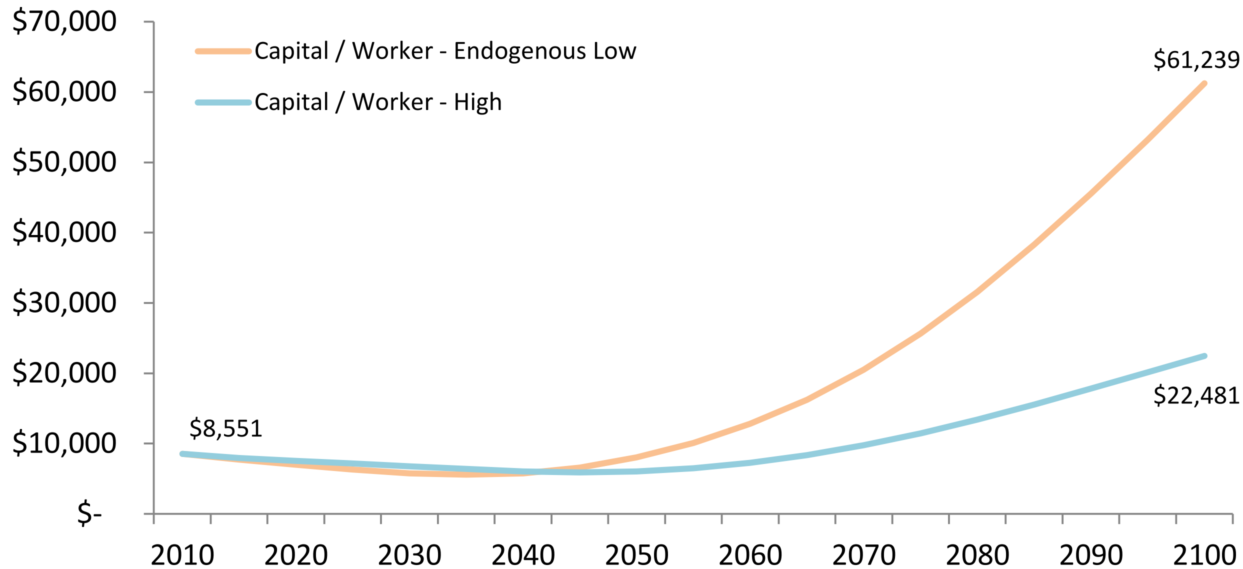


Figure 8 shows the time path of the evolution in modern sector capital per worker, over the 90 year time period. We observe that modern sector capital per worker is fairly stable and approximately equal across both fertility scenarios (and even slightly higher in the high fertility scenario around 2030) until around 2040, after which modern sector capital per worker under the endogenous low-fertility scenario is projected to grow at a faster rate such that the level of modern sector capital per worker in the endogenous low fertility scenario, at an estimated $61,239 per worker, is more than 2.7 times that of the high fertility scenario, at an estimated $22,481 per worker, by the year 2100. However, capital per worker is not set to increase substantially in either scenario for around 50 years. This is because our savings equation is driven to a large extent by the income effect, in which economies with low income levels have low savings rates. It is only when income levels rise substantially that domestic savings take off. This highlights the potential role of foreign investment over the medium run in Sub-Saharan Africa as a source of funds for investment, given the weak initial rates of domestic savings.

5.2 Component Channels and their Long Run Paths

Figures 9 to 12 illustrate the evolution paths of four key component channels through which changes in fertility affect income per capita and other indicators of economic growth in our model. As was the case for the previous graphs, each of these figures present projected paths under the three fertility scenarios. These component channels include:

1. *The working age population ratio*, which is defined as the ratio between the total number of workers in both sectors and the total population at each time period. This measure is a reflection of the potential for a demographic dividend in that it captures the additional productivity that can be generated through mechanical shifts in the population age structure, which in turn is a consequence of declining fertility.
2. *The average years of schooling attained*, which accounts for the education-as-human-capital pathway through which declining fertility contributes to economic growth and productivity.
3. *Average adult height*, which proxies for health as the other human capital pathway in the model.
4. *Female labor force participation*, which reflects the direct labor market opportunity cost of childbearing.

Figure 9: Proportion of Population of working age under high-fertility and endogenous low-fertility variant scenarios, Nigeria

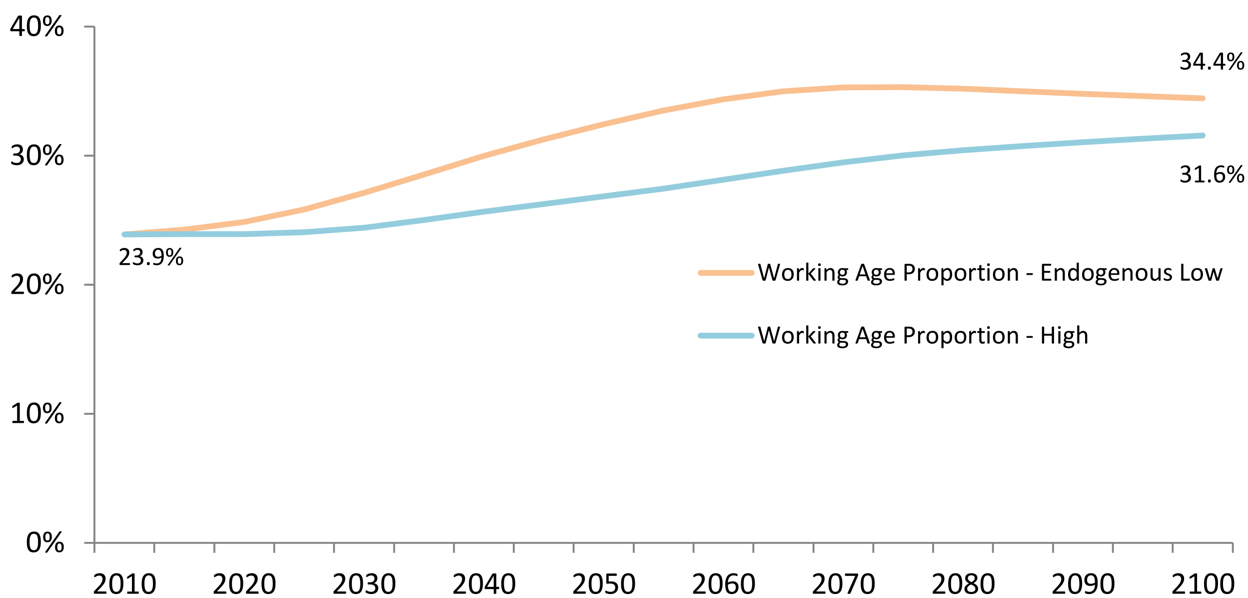


Figure 9 presents the long run effects of declining fertility on the ratio of the working age population (ages 20 to 64) to the total population. Reductions in the fertility rate over time contribute to a higher working age population ratio as the base of the population pyramid shrinks relative to the productive working ages. Moreover, the working age population ratio increases faster with larger declines in fertility. In particular, we see that the difference in fertility between the high fertility scenario and the endogenous low fertility scenario translates to a 6 percentage point difference in their working age population ratios by 2060 and a 2.8 percentage point difference in their working age population ratios by the end of the projection period. A key difference is that lower fertility only increases the working age share of the population until around 2070, after which the sustained low fertility contributes a rise in the old age dependency rates and a falling working age share. While the working age share of the population is always higher under the low fertility variant, the gap is only increasing until 2070. We therefore expect to see most of the income gains in that period, with little additional benefit from age structure after 2070.

Figure 10: Average years of schooling of the workforce under the high-fertility and endogenous low-fertility variant scenarios, Nigeria

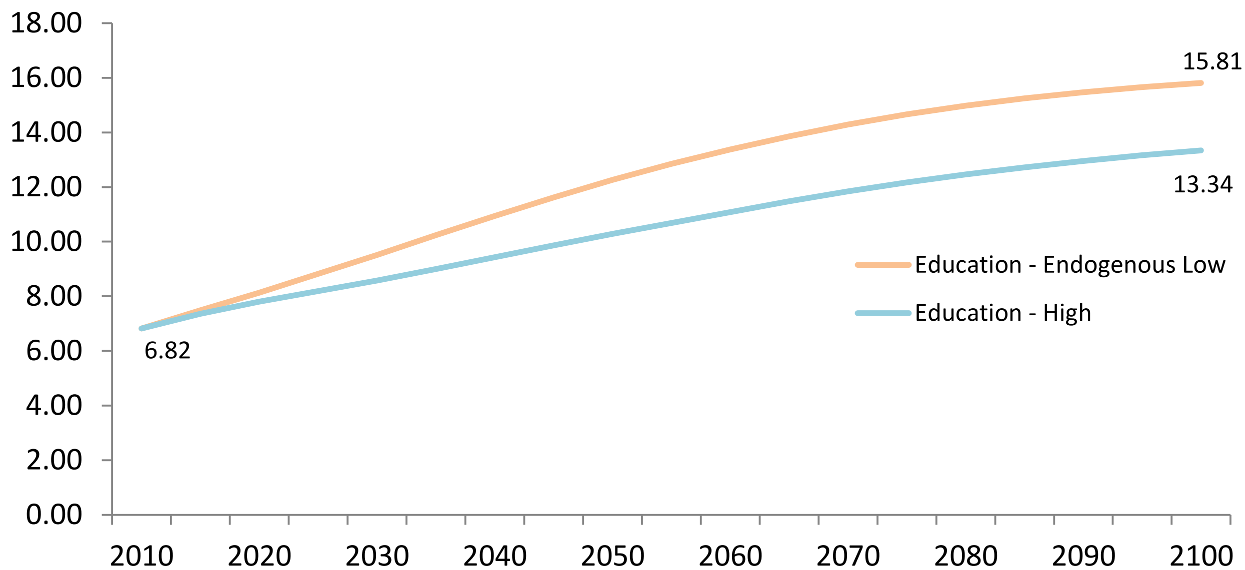
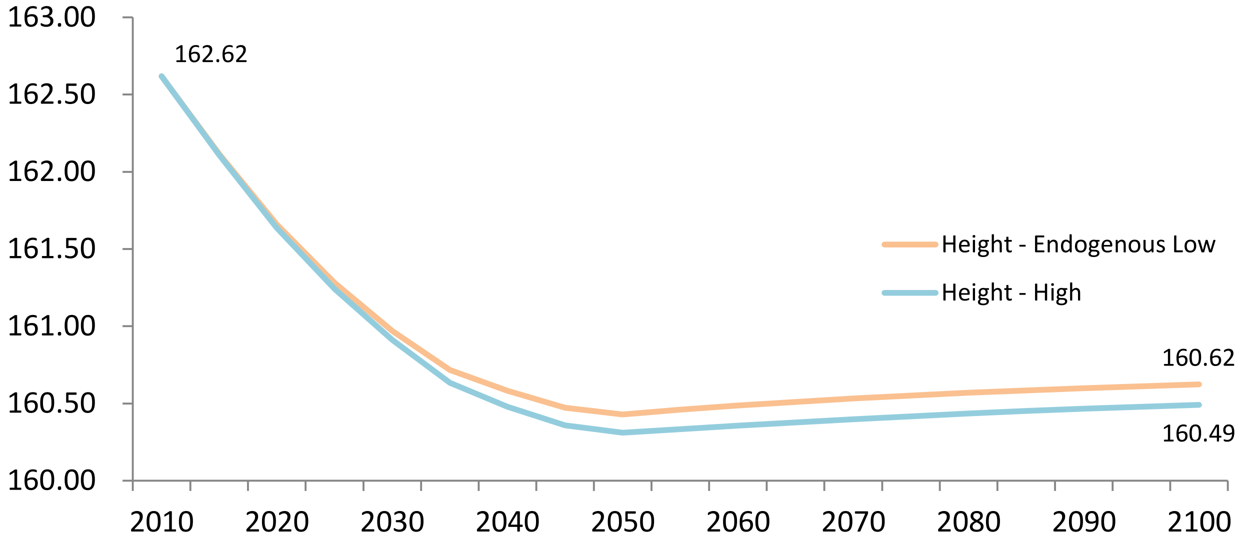


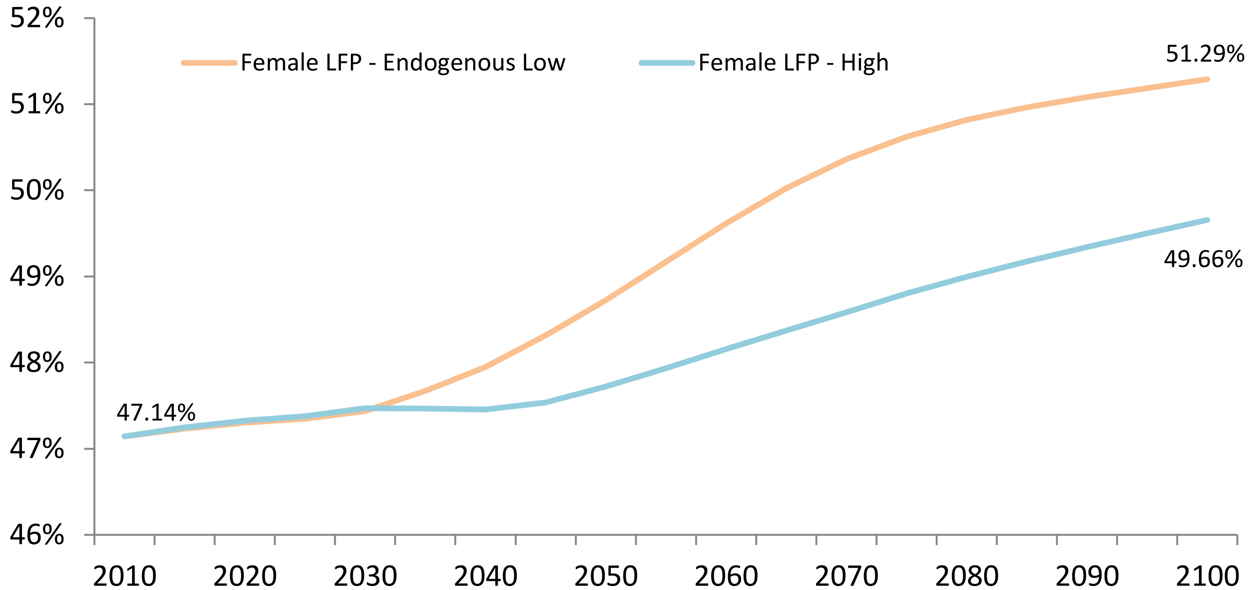
Figure 10 outlines the paths of education, as measured by average years of schooling attained of the workforce, under the two fertility scenarios over the 90 year projection period. This is calculated by using the age and sex specific levels of schooling and labor force participation rates. While educational attainment is expected to increase in the workforce as a whole, it will increase at faster rates under lower fertility. In particular, the average number of schooling obtained by the population under the endogenous low fertility scenario is projected to be 2.47 years more than the average amount of schooling obtained under the baseline high fertility scenario.

Figure 11: Average height of the workforce (in cm) under the high-fertility and endogenous low-fertility variant scenarios, Nigeria



Despite declining infant mortality rates, adult heights have not increased in Sub-Saharan Africa and have even declined in many Sub-Saharan African countries. This conflicting trend is likely due to the fact that infant mortality decline in Sub-Saharan Africa have been brought about by health interventions that target child survival but do little to improve morbidity or child physical development (Akachi & Canning, 2010). Younger cohorts in Nigeria are shorter than those born earlier and we project the average height of the workforce will decline. We therefore find that average adult height in the workforce is projected to decrease in the baseline high fertility scenario over the next 30 years until around 2040, after which adult height stabilizes and starts to increase by the end of the projection period. Similar to education, we predict that health human capital, as proxied by adult stature, will also be higher over time in the endogenous low fertility, as is shown in Figure 11. In particular, adults under the endogenous low fertility scenario are predicted to gain 0.13 cm more than adults under the baseline high fertility scenario over the projection period.

Figure 12: Female labor force participation rate under the high- and endogenous low-variant scenarios, Nigeria 2010



In assessing the female labor supply response to declines in fertility over time, we observe a modest difference over time in labor force participation rates associated with the different fertility scenarios, as is depicted in Figure 12. Most notably, we observe a 1.63 percentage point difference when comparing the female labor participation rate in the endogenous low fertility scenario to the female labor participation rate in the baseline high fertility scenario over the projection period.

5.3 Mechanism Analysis

A decline in fertility, and any subsequent changes in population size and age structure, is likely to affect economic outcomes through several different mechanisms, each of which may operate at a different relative intensity and at a different time horizon (Ashraf et al., 2013). We decompose the overall effect of fertility reduction into the parts that run through these different mechanisms, and we acknowledge there are clearly interactions among the different effects in our model. We perform all of our comparative analyses of the effects of fertility under the assumption that all of the other mechanisms are operative; that is, we relate the results in our fully specified simulation model relative to results from a model in which one mechanism is suppressed[4].

[4] An alternative would have been to conduct a comparative analyses using the model in which no other mechanisms are operative (which, in fact, would be equivalent to a re-simulation of the Ashraf et al. (2013) one sector model) as the point of reference.

We begin by assessing the impact of our four new mechanisms (the use of a three-sector economic framework with market frictions, the inclusion of an endogenous fertility mechanism, the inclusion of health as human capital, or the inclusion of an endogenous savings mechanism) individually on income per capita, which is our main economic outcome of interest. This analysis allows us to determine the sensitivity of our results relative to our assumptions about key parameters. We then fully decompose the effects of each of the mechanisms on income per capita to identify their relative importance at different time horizons.

Figure 13 projects the ratio of income per capita in the endogenous low fertility scenario to income per capita in the high fertility scenario in our full three-sector demographic-economic model, hereafter referred to as the CKW model, across the 90-year time horizon. The figure also compares the CKW results to projections of the income per capita ratio from:

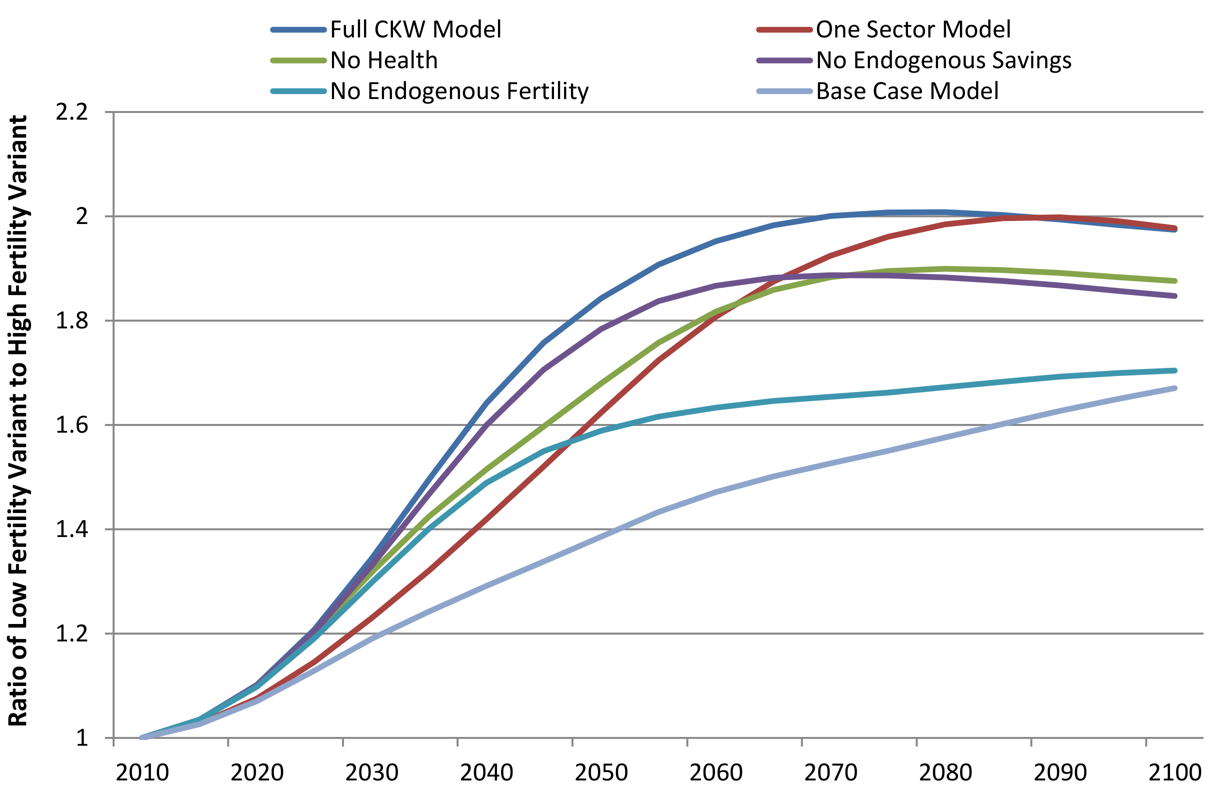
1) alternative models in which one of the four key mechanisms from the CKW model is suppressed; and
2) a simulated "base case" one sector model in which all four key mechanisms from the CKW model are jointly suppressed. This base case model creates a point of comparison for our results by replicating the conditions and results of the Ashraf et al. (2013) model. [5]

Under the CKW model, we find that the long-run effect of reducing fertility from the baseline high variant fertility scenario to the endogenous low variant fertility scenario leads to an 95.2 percent increase in income per capita by 2060, which is roughly double the size of the 47.3 percent increase in income per capita as predicted by the simulated base case model over the same 50-year time horizon. The gains in low fertility occur in our model over a sixty year period with income per capita rising to about twice the level in the high fertility variant and then stabilizing at that higher ratio. The income effects in our model are larger and faster to occur than those that are predicted by the base case model. It is important to note that while the age structure effects of the demographic transition are transitory the human capital effects of moving to low fertility are permanent.

[5] It is important to note that the CKW model in which all four key mechanisms are suppressed differs from the original Ashraf et al. (2013) model only due to differences in parameter values and functional forms between the two models. In this manner, our base case model creates an appropriate counterfactual simulation of income per capita paths under what an Ashraf et al. (2013) model with updated functional specifications and parameter values would have predicted.

Interestingly, we see that the projected path of income per capita under the model where the use of the three-sector economic framework was suppressed (the red line) eventually converges to the projected path that is predicted by the full CKW model over the entire time horizon. Eventually, almost all workers are in the modern sector and the economy essentially collapses into a one sector model with very small contributions from the traditional and natural resource sectors. However, economic growth under the three sector model is much faster than in the one sector model for a considerable period as slower population growth allows for more rapid industrialization and as a higher share of the workforce is absorbed into the modern sector. The effect of endogenous savings is quite small and only really occurs after 2050 when income levels are high enough to make saving feasible. Health makes a contribution that is similar in magnitude to that of savings but at a slightly faster rate. However, the major reason for the difference between the predictions of our model and the predictions from the one sector reference model is the fertility feedback. If we switch this mechanism off, the gains from low fertility occur more quickly but converge to around the same level as the gains predicted by the one sector model. The feedback acts as a multiplier effect increasing the long run gains from any initial level of fertility reduction.

Figure 13: Comparison of income per capita scenarios across models, Nigeria



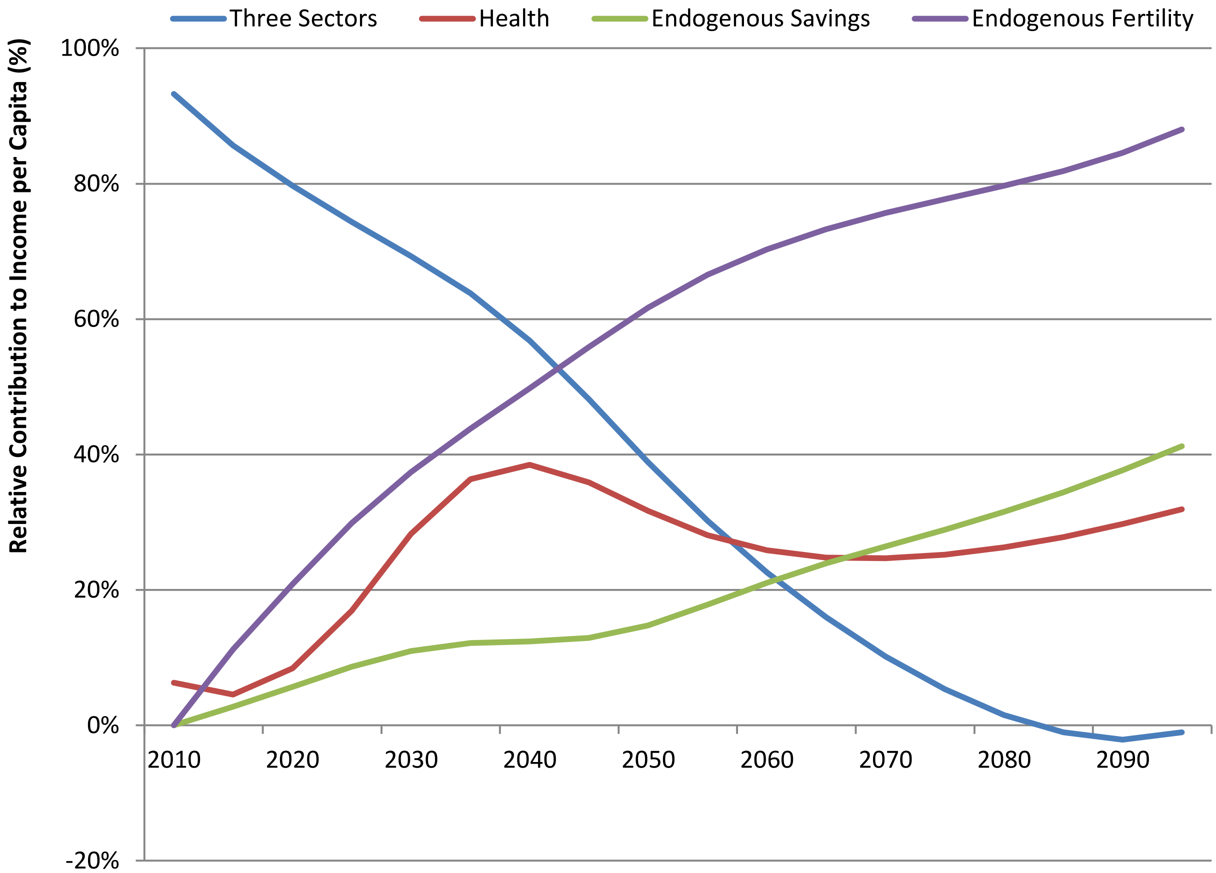
5.4 Decomposition of Mechanisms

Figure 14 presents a full decomposition of the fraction of the gain in income per capita at each point in time that is attributable to the four different mechanisms that are incorporated in the full CKW model. To assess the fraction of the gain in income per capita that is due to each mechanism, we compare the level of income per capita in each year in the CKW model to the level of income per capita that is predicted when the mechanism is suppressed. For each year, we then sum these individual mechanism effects to obtain an estimate the total effect that ignores interactions. Due to the interactions among the mechanisms in the model, the effects from the individual mechanisms do not sum exactly to the total effect of a decline in fertility on income per capita. Finally, we divide the individual effects by the estimated total effect to produce a share of the total income gain attributable to each effect at each point in time and over the 90-year time horizon.

The figure shows that at the start of the projection period, the inclusion of three sectors with economic frictions, accounts for more than 93 percent of the total income gain in the short run, and is by far the dominant mechanism. However, the relative contribution of the three-sector mechanism falls quickly over time to about 30 percent after 50 years. The low fertility allows for a larger percentage of workers to enter the high productivity modern sector and promotes rapid economic growth. Eventually, when almost all of the workers are absorbed into the modern sector, this mechanism becomes unimportant, but sectoral shifts are an important driver of potential growth and accounted for a large part of the economic "miracle" is Asia (Nelson & Pack, 1999; Stiglitz & Yusuf, 2001).

The endogenous fertility multiplier becomes increasing important over time and is the largest contribution to the gains in the long run, accounting for more than two-thirds of the projected income gain over and above that of the simulated one sector base case model by 2060. The rest of the gains in the long run are due to the health and endogenous savings mechanisms.

Figure 14: Decomposition of the gain in income per capita relative to the base case model by mechanism, Nigeria



6. Conclusions

In this paper, we estimate the effect of a decline in fertility on economic growth in Nigeria using a demographic-economic macrosimulation model. We improve on previous modeling approaches, particularly that by Ashraf et al. (2013), to incorporate four previously ignored channels: 1) the effect of fertility on savings; 2) a feedback from education back to fertility; 3) the effect of fertility on health; and 4) the effect of a more realistic three-sector model with market imperfections, which are prevalent in the developing world.

Since the purpose of our paper is to provide a more comprehensive understanding of the relationship between fertility decline and income growth, a natural question to ask is how the additional channels that we add change the results that have previously been found in the literature. We find that adding these additional channels means that lowering the total fertility rate by one child per woman almost doubles income per capita by 2060 which is twice as big as the effect found by Ashraf et al. (2013). We conclude that these previously ignored channels are not only important, but perhaps are even more important than the more traditional channels that have been considered in the literature to date. In the short to medium run, we find the main reason for the higher income effects in our model are due the larger share of the workforce that moves into the modern sector of the economy when fertility is low. In the long run, we find that lower fertility increases female education, which in turn lowers fertility in the next generation and producers a multiplier effect from any initial change in fertility.

In putting our findings into context, we recognize that while our projections for the effects of fertility reduction are roughly double than what has previously been predicted, our estimated effects are generated by relatively large reductions in fertility (one birth) over a relatively short period of time (15 years). While such significant declines in fertility have been observed in contexts where strong family planning programs have been implemented, there is a question whether it is realistic to assume that the same types of programs and policies can replicate such results in Sub-Saharan Africa, where ideal family size and desired fertility are higher than in other parts of the world (Bongaarts, 2011).

In this light, we, like Ashraf et al. (2013) and others, acknowledge that the economic growth brought about by fertility decline would not be sufficient to help a developing country "vault into the ranks of the developed" (National Research Council, 1986). With that said, we argue that asking whether fertility decline could singlehandedly determine a country's path to economic growth and development was never an appropriate question to begin with. It is clear that there are many determinants of economic growth, and it is equally clear that demographic change brought on by a reduction in fertility is one of these determinants. We would highlight institutional factors, such as good governance, a market based economy, openness to international trade, public investment in infrastructure and education, and improvements in total factor productivity, as additional important mechanisms in a holistic view of economic development. Even if fertility were to decline and income per capita were to roughly double as we predict, it would still not be enough to close the estimated 30-fold gap in income per capita between rich and poor countries – to close such a gap would require several doublings in income per capita (United Nations Conference on Trade and Development, 2002). However, while not the whole story, our model suggests that reducing fertility can make a substantial contribution to economic development in Africa.

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Appendix 1: The Model

Population

We take a baseline age structure together with age-specific mortality and fertility rates to project the population over time. We divide population is divided into 21 age groups indexed from $i = 0,1, \ldots ,20$, with each group covering a 5-year age interval for populations aged 0-104 years of age and each time period t in our model corresponding to five years. The population at time t in age group i, $Pop_{i,t}$ for $i \geq 1$ is given by

$$Pop_{i,t} = \left(1 - d_{i,t}\right) Pop_{i-1,t-1}$$

where $d_{i,t}$ refers to the mortality rate for age group i in period t and $\left(1 - d_{i,t}\right)$ is the proportion of the cohort surviving into the next age group in in the next period. We assume that $d_{20,t} = 0$ so that no one survives to age 105. The population in age group 0, i.e. those who are between age 0 to 4 years of age, is given by the age group specific fertility rate in the period multiplied by the size of the that age group of female population, $Popf_{i,t}$, for women of reproductive age (ages 15 to 49). We also allow for infant and child mortality to be given by $d_{0,t}$, which is the proportion of births not surviving to be measured in the 0 to 4 year age cohort.

$$Pop_{0,t} = \left(1 - d_{0,t}\right) \sum_{i=3}^{9} f_{i,t}\, Popf_{i,t}$$

We follow the age structure of the female and male population separately from the initial year but assume that in future birth and death rates are the same for each sex.

We adopt the UNPP projections on mortality rates over time for each fertility scenario, which implies that age group specific mortality rates in each future five-year period are assumed to be fixed across each of the two fertility scenarios, but are not fixed over time, throughout our analysis. Each age group specific mortality rate is calculated as the implicit age group-by-year mortality rate from the medium fertility variant projection the United Nations World Population Prospects 2012. For example, the implicit death rate in age group 0 for both the baseline and alternative fertility scenarios is given by the population aged 0 to 4 divided by the number of births in the 5 year period under the medium variant fertility scenario.

On the other hand, we deviate from the UNPP methodology in our calculations of fertility across each scenario and over time. In contrast to previous simulation approaches, we endogenize the evolution of fertility over time by introducing a feedback mechanism from female education to fertility in a log-linear form as follows:

$$\log f_{i,t} = \log(f_{i,t}^* - \lambda_{i,t}) + \psi(EF_{i,t} - EF_{i,t}^*)$$

where $f_{i,t}^*$ is the fertility of age group i at time t forecasted by the United Nations Population Projection's high variant fertility scenario, $\lambda_{i,t}$ is the estimated age-specific fertility given the effect of the exogenous reduction in fertility (in our case, a family planning intervention) from the baseline high fertility scenario to the alternative low fertility scenario, $EF_{i,t}^*$ is the level of female education, measured by years of schooling, in the baseline high variant fertility scenario, and $EF_{i,t}$ is the level of female education that results given the deviation in fertility from the baseline high variant fertility scenario to fertility under the alternative low fertility variant scenario. The parameter ψ, which we expect to be negative, intends to capture the direct effect of increased female education on fertility. When considering how to measure educational attainment, we choose to use years of schooling as a proxy because it is tractable enough to be estimated using cross-country data and is widely accepted as a standard metric in academic and policy spheres, which in turn allows us to compare our estimates against existing evidence. Nevertheless, we recognize that our choice is limited to the extent that years of schooling may not reflect other key dimensions that determine educational attainment, including education quality, types of educational attainment (vocational training, apprenticeships, etc.), among others. Further refinements to our measure is planned for future work as additional data on these other factors are collected from low- and middle-income countries.

Labor Supply

We assume that children may enter the labor force at 20 and workers leave the labor force at 65. Our rationale for restricting our definition of the working age population to this age range is rooted in evidence from the national transfer accounts literature, which find that over 90 percent of lifetime earnings is accumulated in this age range in both developing and developed economies; moreover, labor income is low for children and young adults under 20, particularly in African economies due to poor employment opportunities, and the share of lifetime earnings at old ages is, at best, modest (Lee & Mason, 2011). For each sex, we calculate the total labor supply contribution at time t as a function of the labor force participation rate at each age group i and time period t, $partm_{i,t}$ for males and $partf_{i,t}$ for females, and the size of the sex-specific population of age i at time t. Total male and female labor supply at time t, LSM_t and LSF_t, respectively, and total labor force LS_t are determined by

$$LSM_t = \sum_{i=4}^{12} partm_{i,t} Popm_{i,t} \qquad LSF_t = \sum_{i=4}^{12} partf_{i,t} Popf_{i,t} \qquad LS_t = LSM_t + LSF_t$$

where $Popm_{i,t}$ and $Popf_{i,t}$ are the projected male and female populations, respectively, in age group i at time t. We assume age specific male participation rates are constant over time and are fixed at their baseline level, $partm_{i,t} = partm_{i,0}$. We then modify the age-specific female labor force participation rate at t to reflect the effect of a decrease in total female labor supply due to increases in time devoted to childrearing, namely

$$partf_{i,t} = partf_{i,0} + \pi(f_{i,0} - f_{i,t})$$

where $partf_{i,0}$ is the baseline female labor force participation rate for age group i, π measures the effect of fertility on female labor supply, and $(f_{i,0} - f_{i,t})$ captures the difference between the age-specific fertility rate for cohort i at time t and the fertility rate of the group in the first five-year interval. Through this equation, we can predict the increase in female labor force participation rates as age-specific fertility rates decline (i.e. as the difference between baseline age-specific fertility rate $f_{i,0}$ and $f_{i,t}$ grows larger and becomes increasingly more positive). In our specification, we assume that there is no selection into labor force participation by either education or health – the human capital of each worker in age cohort is assumed to be equal to the average human capital of the cohort.

Our prediction is that while fertility inversely varies with female labor supply in most developed country settings, the same relationship is found to not be true in the Sub-Saharan African context, where household composition and the division of labor and childcare responsibilities among household members imply that the effect of fertility on female labor supply may be small or even positive (Westeneng & D'Exelle, 2011). Moreover, women in Sub-Saharan Africa tend to be strongly attached to the labor market, working less during pregnancies but returning to the labor market right after giving birth. Because many women in Sub-Saharan Africa are either self-employed or work in the informal sector, mothers can and often do bring their young children to work with them. Hence, female labor force participation in these settings is already high, even during women's reproductive years, and women contribute to the labor force, though mostly through working in the informal sector and for low wages.

Education

We assume that a given sex specific cohort's educational attainment (quantified in average years of schooling per individual) is entirely amassed before age 20, after which the average level of schooling for that cohort is held constant for the remainder of that cohort's lifetime[6]. Letting $EM_{i,t}$ be the education of the male cohort and $EF_{i,t}$ be the education of the female cohort of age group i at time t, we have

$$EM_{i,t} = EM_{i-1,t-1} \text{ for } i \geq 5$$
$$EF_{i,t} = EF_{i-1,t-1} \text{ for } i \geq 5$$

We also expect that lower fertility will raise the average level of schooling. Models of the fertility transition stress the movement of households along a "quality-quantity" frontier in which investment per child in health and education rises as the number of children falls (Becker, 1981; Becker & Lewis, 1973; Lam, 2003). We assume that the cohort's average years of schooling amassed by age 20, denoted $EM_{4,t}$ for male cohorts and $EF_{4,t}$ for female cohorts, is given by:

$$EM_{4,t} = EM^*_{4,t}[1 + \theta_E(TFR_{t-4} - TFR^*_{t-4})]$$
$$EF_{4,t} = EF^*_{4,t}[1 + \theta_E(TFR_{t-4} - TFR^*_{t-4})]$$

where $EM^*_{4,t}$ and $EF^*_{4,t}$ are exogenous measures of the average number of years of schooling acquired by each cohort in the baseline scenario, TFR_t is the total fertility rate at time t calculated from the age specific fertility rates of women of reproductive age in that time period, and TFR^*_t is the total fertility rate at t under the baseline scenario. The equations are specified using local linear approximations of the fertility-education relationship around each cohort's average number of years of schooling in the baseline scenario, $EM^*_{4,t}$ and $EF^*_{4,t}$. The parameter θ_E, which is assumed to be positive, captures the effect of fertility on children's education and is weighted by the cohort's baseline measure of schooling $EM^*_{4,t}$ and $EF^*_{4,t}$ such that a higher cohort baseline level of schooling lead to larger marginal gains to education from changes in fertility.

In our simulation model, we calculate average years of schooling at time t separately for each sex as a weighted sum of the average years of schooling of each cohort, using

[6] In assuming that the stock of schooling remains constant after it is accumulated by age 20, we neglect to adjust for factors that reflect the depreciation of educational attainment over time. These factors, which are also likely to be associated with determinants of education quality, would certainly enrich the scope of analysis but are excluded because they, like educational quality, are difficult to estimate over time and across countries.

$$EM_t = \sum_{i=4}^{12} \left[\frac{partm_{i,t} \cdot Popm_{i,t}}{\sum_{i=4}^{12} \left(partm_{i,0} \cdot Popm_{i,t} \right)} \right] EM_{i,t}$$

$$EF_t = \sum_{i=4}^{12} \left[\frac{partf_{i,t} \cdot Popf_{i,t}}{\sum_{i=4}^{12} \left(partf_{i,t} \cdot Popf_{i,t} \right)} \right] EF_{i,t}$$

and then combine the sex-specific estimates in a weighted average to estimate the average years of schooling for the entire workforce at time t

$$E_t = \frac{EM_t LSM_t + EF_t LSF_t}{LS_t}$$

Health

Our treatment of health parallels our model assumptions on educational attainment and schooling in the previous section. We proxy cohort health by cohort average adult height. Adult height has been found to be sensitive to childhood health and nutrition and is linked in turn to adult worker productivity (Schultz, 2002). We assume that a given cohort's average height is attained by age 20, after which the average height for that cohort is held constant. We expect lower fertility to be reflected in additional investments that households with fewer children are able to make to improve child health and nutrition, which in turn reduce stunting and positively contribute to growth and development into adulthood.

To estimate the effects of fertility on a given cohort's height at time t, we assume that the cohort's average height amassed by age 20, denoted $HM_{4,t}$ for male cohorts and $HF_{4,t}$ for female cohorts, is given by:

$$HM_{4,t} = HM_{4,t}^{*}[1 + \theta_H(TFR_{t-4} - TFR_{t-4}^{*})]$$
$$HF_{4,t} = HF_{4,t}^{*}[1 + \theta_H(TFR_{t-4} - TFR_{t-4}^{*})]$$

where $HM_{4,t}^{*}$ and $HF_{4,t}^{*}$ are exogenous measures of the average height of each cohort in the baseline scenario, and θ_H is an exogenous constant that captures the direct effect of fertility on adult height. These equations mirror the equations that have been used to describe the relationship between fertility and education.

As was the case with our education estimates, we assume that the average height past age 20 for a given cohort i, $H_{i,t}$, remain constant. In particular:

$$HM_{i,t} = HM_{i-1,t-1} \text{ for } i \geq 5$$

$$HF_{i,t} = HF_{i-1,t-1} \text{ for } i \geq 5$$

Similarly, we calculate average height separately for each sex, namely

$$HM_t = \sum_{i=4}^{12} \left[\frac{partm_{i,t} \cdot Popm_{i,t}}{\sum_{i=4}^{12} \left(partm_{i,0} \cdot Popm_{i,t} \right)} \right] HM_{i,t}$$

$$HF_t = \sum_{i=4}^{12} \left[\frac{partf_{i,t} \cdot Popf_{i,t}}{\sum_{i=4}^{12} \left(partf_{i,t} \cdot Popf_{i,t} \right)} \right] HF_{i,t}$$

and then combine the sex-specific estimates in a weighted average to estimate the average height of the workforce at time t

$$H_t = \frac{HM_t LSM_t + HF_t LSF_t}{LS_t}$$

Production

We consider a Lewis development economy with three sectors, a modern sector, a traditional sector, which share the total labor supply across sectors to produce distinct commodities, and a raw materials sector. Aggregate production in the modern sector at time t is given by a standard Cobb-Douglas production function, with physical capital K_t, labor allocated to the modern sector LM_t, average years of schooling in the workforce (as a proxy for education) E_t, and average height of the workforce (as a proxy for health) H_t as factor inputs such that aggregate output in the modern sector at t, YM_t, is given by

$$YM_t = AM_t K_t^{\alpha} LM_t^{1-\alpha} e^{\gamma E_t + \lambda H_t}$$

where AM_t is the total factor productivity of the modern sector at t. Estimates for schooling E_t and health H_t are fed into the economic model from our demographic simulations as described in the previous section.

In a similar fashion, aggregate production in the traditional sector at t is also modeled by a Cobb-Douglas production function, with available land X and labor allocated to the traditional sector LA_t as factor inputs such that aggregate output from the traditional sector at t, YA_t, is given by

$$YA_t = AA_t X^{\beta} LA_t^{1-\beta}$$

where AA_t is the total factor productivity of the traditional sector at t.

Capital Accumulation and Savings

In our model, we extend the standard Solow framework for capital accumulation by assuming that capital stock in the period $t+1$, K_{t+1}, evolves over time according to the equation

$$K_{t+1} = s_t Y_t + (1-\delta) K_t$$

where s_t is the savings rate at time t and δ is the rate of depreciation of capital that is assigned a standard value of 7 percent (Schmitt-Grohe & Uribe, 2006). We depart from the simplifying assumption of a constant savings rate and follow Bloom, Canning, Mansfield, et al. (2007), in which the evolution of the savings rate is defined by

$$s_t = \frac{S_t}{Y_t} = \phi_0 + \phi_1 s_{t-1} + \phi_2 w_t + \phi_3 w_t^2 + \phi_4 \frac{Old_t}{WA_t}$$

Here, $s_{t-1} = \frac{S_{t-1}}{Y_{t-1}}$ is the savings rate in the previous time period $t-1$, w_t is the annual aggregate wage at time t, which is defined as a fixed proportion of per-capita income in the same period (i.e. $w_t = (1-a) y_t$ for some fixed a), and $\frac{Old_t}{WA_t}$ captures the old-age dependency ratio, the ratio of old-age dependents to the working age population, at t. We assume that savings begins in a steady state equilibrium in 2010, that generates the observed capital stock, and we calibrate the constant term ϕ_0 to fit the baseline steady state savings, wage, and dependency ratio conditions. Further details on the derivation and interpretation of the savings equation can be found in Appendix 2.

Labor Allocation across Sectors

Our model specification requires that modern sector and traditional sector wages, which endogenously adjust within their respective labor markets, will in turn determine equilibrium labor supply allocations across the two sectors that employ workers. Total labor supply L_t is shared across the modern and traditional sectors such that

$$L_t = LM_t + LA_t$$

The wage per worker in the modern sector at time t, wM_t, is set to be equal to the marginal product of labor in the modern sector for an additional worker with average levels of education and health, or in log terms

$$\log wM_t = \log\left[(1-\alpha)\frac{YM_t}{LM_t}\right]$$

In contrast, we assume that the traditional sector is less developed and is more labor intensive with little to no capital endowment, thereby resulting in the wage per worker in the traditional sector at time t, wA_t, being determined by the average product, or in log terms:

$$\log wA_t = \log\frac{YA_t}{LA_t}$$

Since the wage in the traditional sector is determined at the average and not on the margin, in equilibrium there will be too many workers in the traditional sector. In addition, there may be migration costs or other barriers to entry into modern sector jobs, which are parametrized by the term b, that will contribute to an inefficient allocation of labor across sectors. In equilibrium, workers will migrate between sectors and wages will adjust such that

$$\log wM_t - \log b = \log wA_t$$

Here, b is a constant that is set so as to explain any baseline differential in sector wages and is then held constant over time. If we replace modern sector and traditional sector wages with their respective wage-output equilibrium conditions and substitute modern sector and traditional sector output with their respective production functions, we obtain:

$$Z_t LM_t^{-\alpha} = (L_t - LM_t)^{-\beta}$$

where

$$Z_t = \frac{(1-\alpha)\cdot AM_t K_t^{\alpha} e^{\gamma E_t + \lambda\, H_t}}{b \cdot AA_t X^{\beta}}$$

For $\alpha = \frac{1}{3}$ and $\beta = \frac{1}{6}$, we can explicitly solve for LM_t as

$$LM_t = \frac{1}{2}\left(Z_t^3\sqrt{Z_t^6 + 4L_t} - Z_t^6\right)$$

We can verify that $0 \leq LM_t \leq L_t$, and we calibrate the value of b so that initial labor stock in the modern sector, LM_t, matches the data. We then fix b to that value in all subsequent simulations.

Appendix 2: The Savings Equation

In modeling the evolution of savings, we follow the example of Bloom, Canning, Mansfield, et al. (2007) in which we consider cohort-specific savings decisions over time and aggregate across cohorts to find national savings. In the Bloom et al. (2007) savings model, the authors allow for both retirement decisions and savings decisions to depend on life expectancy, in which they argue that longer life spans lead to longer periods of retirement and increased pre-retirement savings. To derive the savings relationship, the authors first jointly solve for individuals' optimal lifetime labor supply, consumption, and savings, which are functions of life expectancy, using a lifetime utility maximization problem and derive the aggregate savings relationship (Equation 30) as follows:

$$s_t = \frac{S_t}{Y_t} = h(z, \sigma, w_t, R^*) + \frac{\sigma}{BR} - \frac{Old_t}{WA_t} + \eta \frac{Young_t}{WA_t} + \log\left(\frac{LF_t}{WA_t}\right) + \log(1-\alpha)$$

where z is life expectancy, σ is the growth rate of wages, w_t is the wage rate at time t, R^* is a mandatory retirement age constraint (usually 65), BR is the birth rate, $\frac{Old_t}{WA_t}$ captures the old-age dependency rate at t, $\frac{LF_t}{WA_t}$ captures the labor force participation rate at t, and α is the capital share of output.

To estimate the equation above, the authors test for potential non-linear effects of life expectancy, wages, and wage growth rate on savings behavior by performing a second-order Taylor series expansion on the h function around these three variables and including first-level interaction terms in h. They also include a lagged savings rate term to adjust for dynamic dependency in the time path of savings. The parameters of this saturated equation are then estimated in a dynamic fixed effects panel model using data for a panel of countries from 1960 to 2000 and a specification that is robust to country fixed effects and that allows for a dynamic evolution of aggregate savings as it adjusts towards its steady state (Table 4, Column 3). After removing insignificant variables sequentially, the authors arrive at the final regression specification below (Table 4, Column 4), which we use as our main savings equation:

$$s_t = \phi_0 + \phi_1 s_{t-1} + \phi_2 w_t + \phi_3 w_t^2 + \phi_4 \frac{Old_t}{WA_t}$$

To parameterize the coefficients ϕ_1 to ϕ_4 in this specification, we use estimates from the full model in Table 4, Column 3, and we then calibrate the estimate for ϕ_0 to be the value that achieves a steady state rate of savings under the baseline conditions for savings, wages, and the age dependency ratio, i.e. ϕ_0 is fit under $s_t = s_{t-1} = s^*$, the steady state savings rate, for the given s_0, w_0, and $\frac{Old_0}{WA_0}$.