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Income, Residential Location, and Mode Choice*

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Commuters choose a mode, defined in terms of trip speed and time, to minimize the money and opportunity costs of travel. New results: the wage-rate elasticity of marginal commuting cost may be zero, negative, greater than one, or declining with the wage rate; a constant wage-rate elasticity is possible only if the marginal money cost of commuting is zero; a positive value of the marginal money cost of commuting implicitly assumes mode choice. The relationship between income and location seems due largely to technological change in intraurban transportation, differences in the money cost of commuting by alternative modes, and differences in real wages.

Muth [15] showed that the relative magnitudes of the wage-rate elasticity of marginal commuting cost and the wage-rate elasticity of housing demand determine the income–location relationship in a partial equilibrium model of urban household behavior.[1] In symbols,

$$\frac{\partial k}{\partial w} \begin{matrix} > \\ = \\ < \end{matrix} 0 \text{ as } \epsilon_{q,w} \begin{matrix} > \\ = \\ < \end{matrix} \epsilon_{\mathrm{MC},w}, \tag{1}$$

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[1]Although this paper adheres to the Muthian tradition, we, of course, do not deny that many factors other than the tradeoff between housing expenditures and commuting cost affect residential location.

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where k is distance from the CBD, w is the wage rate, $\epsilon_{q,w}$ is the wage-rate elasticity of housing demand, and $\epsilon_{\mathrm{MC},w}$ is the wage-rate elasticity of marginal commuting cost.

In Muth's original model and its descendants, however, there is no mode choice, and the wage rate affects commuting cost only through the value of time. For example, Muth assumed the total cost of commuting to be given by $T = T(k, y)$, where k is as defined above and y is income. He assumed $T_k > 0$ and $T_{kk} \leq 0$; i.e., commuting cost increases at a nonincreasing rate with distance traveled.[2] To capture the opportunity cost of commuting in terms of lost leisure, Muth included income in the commuting cost function and assumed $1 > T_y > 0$, i.e., that commuting cost increases with income but not by the full amount of the income increase. In addition, he assumed that $T_{ky} \geq 0$. If income increases due to an increase in the wage rate, then $T_{ky} > 0$; if due to an increase in nonwage income, then $T_{ky} = 0$. Similar assumptions about commuting cost are used by other writers (for a survey, see [17]).

Although Muth's model does not explicitly incorporate commuting time, in models that do, such as [4, 18, 19], both the money cost of commuting and commuting time are solely functions of distance.[3] Consequently, in most models of urban household behavior, the wage rate affects commuting cost through the value of time, but it does not affect the commuter's choice of transport mode nor the money cost of commuting.

Casual observation reveals that in U.S. cities lower income individuals commute predominantly by bus while higher income individuals commute predominantly by car. Thus, both mode choice and the money cost of travel seem to be influenced by income. In addition, over time, as real income has risen, the dominant mode of intracity transit has changed. In the United States, prior to 1830, walking was the predominant mode of commuting. From about 1830 to 1850, the omnibus (a horse-drawn vehicle carrying 12 passengers) and commuter railroads became available. Starting in the 1850s, the horse-drawn (and, later, electrified) rail-guided streetcar was introduced. Finally, this mode was supplanted by the automobile. (For further elaboration on these developments, see [13].) As LeRoy and Sonstelie [13] note, income affected both the choice of mode and its money cost.

[2] Here and throughout the paper, subscripts on functional operators denote differentiation.

[3] Hekman's [10] treatment is an exception. In his model, the husband's commuting time is independent of the wage rate, while the wife's depends on her labor supply (hours worked). The household's money cost of commuting depends on distance (assumed the same for both husband and wife) and the wife's labor supply. Since the wife's labor supply is affected by both her and her husband's wages, then both the wife's commuting time and the household's money cost of commuting are indirectly affected by the husband's and wife's wage rates. Mode choice is not considered, however.

To explore the implications of these observations, we develop a model of mode choice in the context of urban commuting. We show that, in such a model, $\epsilon_{MC,w}$ may vary with w in unexpected ways, and that it may not even be positive. Using these results, we then examine the model's ability to explain real-world income−location patterns in urban areas.

1. THE MODEL

A. *Introduction*

Most who have dealt with mode choice, such as [1−3, 8, 13], have treated modes as discrete entities. Instead, we define a mode in terms of its average speed and associated travel time, both continuous variables. Our approach is in the spirit of Quandt and Baumol's [16] abstract-mode concept:

> An abstract mode is characterized by the values of the several variables that affect the desirability of the mode's service to the public: speed, frequency of service, comfort, and cost. Thus, one can define a continuum of abstract modes; most of them may have no current counterpart although some of them will become realities in the future. [16, p. 14]

As concrete examples, Quandt and Baumol used travel time, round-trip cost, and number of departures per day (they emphasized intercity travel) as the set of variables characterizing a mode. We shall use speed, time, and their associated cost.

Treating transportation modes in terms of continuous variables has advantages over the more natural assumption of discrete modes. The first advantage is convenience. We can incorporate mode choice into a partial equilibrium model relatively easily, as will be seen below. Continuity also allows us to use calculus, which keeps the mathematics simpler and more easily interpreted than might otherwise be the case. Another advantage is that we can deal with commuting modes no longer widely used, such as walking, omnibus, and streetcars, as yet undiscovered modes, and those existing modes not now economically feasible, such as electric vehicles. These modes can be handled as easily as familiar and currently used modes. Thus, for convenience and for the ability to deal with long-term changes in transportation technology, representation of modes as continuous, rather than discrete, choices seems justified.

Commuting time, average commuting speed, and distance are related by the equation $k = st$, where k is as previously defined, s is average trip speed, and t is trip time. We define the money cost of commuting as $E(s, t) = F(s) + V(s)t$. The total money cost of commuting, $E(s, t)$, is a function of both speed and time and consists of a component independent of time, $F(s)$, and a component that varies with time, $V(s)t$. We shall refer to the former as fixed cost and the latter as variable cost.

$F(s)$ may have a component that is independent of both time and speed, which would typically be called fixed cost. Consequently, $F(s)$ is fixed in the sense that once an average speed, which indexes a mode, is chosen, this cost component does not change with travel time. This usage does have the drawback, as one referee noted, that a Ferrari and a Hyundai with the same average commuting speed would have the same F. While this is so, we are thinking of broad classes of modes, such as bus and auto, not alternatives within a given mode, such as specific models of buses and cars. When considering the various modes typically used for commuting, either now or in the past, it seems reasonable to array them in terms of their average speeds.

The term $V(s)t$ implies that $E(s,t)$ rises with t at a constant rate, $V(s)$, e.g, one more hour's driving adds the same amount to cost as the previous hour did. This seems to us a reasonable assumption, although it is possible to think of exceptions; e.g., even at a constant speed, depreciation due to time may be higher in the second hour of driving than in the first.

We assume that $E_s > 0$, i.e., it costs more to go faster. Although our formulation is general enough to encompass a single mode (it requires more gasoline or a more powerful car to go faster), we have in mind alternative modes with different average commuting speeds, such as bus and auto. We make no assumptions about the signs of V_s or F_s, except that their signs are such that $E_s = F_s + V_s t$ is positive. Although it might seem reasonable to suppose both F_s and V_s positive, that would not help us obtain any more comparative-static results than we already do without the sign assumptions.

The opportunity cost of commuting is given by wt. This follows from a leisure-choice model in which leisure is the only time argument of the utility function and there are no constraints on work time. Under these conditions, the marginal value of commuting time equals the wage rate in equilibrium.[4] Thus, the marginal opportunity cost of time is the wage rate, and the total opportunity cost of commuting time is wt. Total commuting cost is given by $C = E(s,t) + wt$.

[4] See, for example [4]. Nothing of substance for the purpose of this paper is lost by letting the marginal cost of time spent commuting equal the wage rate. If we allow for preferences for work time and commuting time, then $(u_L/u_x) - (u_W/u_x) = w$, where L is leisure time, x is nonhousing, nontransportation expenditures, and W is work time. The marginal cost of time spent commuting is $(u_L/u_x) - (u_t/u_x)$, or, given the preceding result, the marginal cost of time spent commuting is $w + (u_W/u_x) - (u_t/u_x)$. Constraints on work time would also cause a divergence between the marginal cost of time spent commuting and the wage rate. Empirically, the marginal cost of time spent commuting is less than the wage rate. (The most recent review of which we are aware is [11].) Theoretically, it is usually assumed to be an increasing function of the wage rate (as, for example, in [15]) or a constant fraction of the wage rate (as, for example, in [5]). The results of this paper hold if either of these assumptions is made.

Although a complete analysis of household behavior that incorporated mode choice would involve the specification and comparative-static analysis of a budget-constrained and time-constrained utility maximization model, it is unnecessary for our purposes to include that analysis here. In the "standard" model, the commuting cost function, e.g., Muth's $T(k, y)$, is exogenous, while our contribution is the derivation of that function and the investigation of its properties. Once the function is derived, however, the analysis of the utility maximization model is little different from that of the standard case.[5]

B. *A Cost Minimization Model of Mode Choice*

The problem is to choose s and t to minimize $C = E(s,t) + wt$ subject to $k = st$. Setting this problem up in Lagrangean form, we have

$$\mathscr{L} = E(s,t) + wt + \phi(k - st). \tag{2}$$

First-order conditions are

$$\partial\mathscr{L}/\partial s = E_s - \phi t = F_s + (V_s - \phi)t = 0 \tag{3}$$

$$\partial\mathscr{L}/\partial t = V + w - \phi s = 0 \tag{4}$$

$$\partial\mathscr{L}/\partial\phi = k - st = 0. \tag{5}$$

The second-order condition is

$$D = \begin{vmatrix} E_{ss} & V_s - \phi & -t \\ V_s - \phi & 0 & -s \\ -t & -s & 0 \end{vmatrix} < 0, \tag{6}$$

where $E_{ss} \equiv F_{ss} + V_{ss}t$.

An analogy to production theory may facilitate interpretation of the model. $k = st$ plays the role of the production function, and $E + wt$ plays the role of the cost equation. From the first two first-order conditions, we get the relationship

$$t/s = E_s/(V + w). \tag{7}$$

Equation (7) says that at the cost-minimizing t–s combination, the marginal rate of technical substitution of s for t equals the ratio of the input "prices." The choice of the optimal t–s combination defines a mode of transportation in this model.

[5] We have developed such a model, and it is available as an unpublished appendix to the present article upon request to either author. In addition to a complete comparative-static analysis, we provide a derivation of the text's Eq. (1) in the context of that model. The results are essentially the same as those of [4].

Isoquants have the usual properties assumed in production theory. From $k = st$, $dt/ds = -t/s < 0$ and $d^2t/ds^2 = 2t/s^2 > 0$, so the isoquant is negatively sloped and convex.

The isocost line, however, is somewhat more complex than in standard production theory. Our isocost line may be written as

$$t = \frac{C}{V+w} - \frac{F}{V+w}. \tag{8}$$

Hence, its slope is given by

$$\frac{dt}{ds} = -\frac{E_s}{V+w} < 0, \tag{9}$$

and the rate of change of its slope is given by

$$\frac{d^2t}{ds^2} = -\frac{E_{ss}}{V+w} - \frac{2V_s(dt/ds)}{V+w} \begin{matrix} > \\ = \\ < \end{matrix} 0 \qquad \text{as } E_{ss} \begin{matrix} < \\ = \\ > \end{matrix} -2V_s(dt/ds). \tag{10}$$

The isocost line is convex if $E_{ss} < -2V_s(dt/ds)$, linear if $E_{ss} = -2V_s(dt/ds)$, and concave if $E_{ss} > -2V_s(dt/ds)$.

The second-order condition requires the isoquant to be more convex than the isocost line. To see this, expand the determinant, D,

$$D = 2st(V_s - \phi) - s^2E_{ss} < 0. \tag{11}$$

Rearranging Eq. (11), so that the rate of change of the slope of the isoquant, $d^2t/ds^2 = 2t/s^2$ is on the left-hand side produces

$$\frac{2t}{s^2} > \frac{2V_s(t/s) - E_{ss}}{s\phi}. \tag{12}$$

Then, substituting $V + w$ for ϕs from the first-order conditions, and recognizing that in equilibrium the slope of the isoquant (t/s) equals the slope of the isocost line (dt/ds), we get

$$\frac{2t}{s^2} > -\frac{E_{ss}}{V+w} - \frac{2V_s(dt/ds)}{V+w}. \tag{13}$$

The right-hand side of Eq. (13) is the rate of change of the isocost line's slope as given in Eq. (10).

Before turning to the comparative statics of the model, we want to note another implication of the first-order conditions. Using the Envelope

Theorem and the first-order conditions, we have

$$\partial\mathscr{L}/\partial k \equiv \partial C/\partial k \equiv \mathrm{MC} = \phi = [V(s) + w]/s = E_s/t > 0, \quad (14)$$

where C is the optimized cost function. Thus, marginal (distance) commuting cost is positive. Note that ϕ, the Lagrange multiplier, measures the sensitivity of C (commuting cost) to changes in the k constraint (distance). It is the wage-rate elasticity of this marginal cost, $\epsilon_{\mathrm{MC},w}$, that plays such an important role in the effect of income on location.

C. *Comparative-Static Analysis of the Model*

For purpose of the comparative statics, we could introduce shift parameters into the fixed money cost of speed function and the variable cost of speed function, but since the comparative-static results involving changes in these functions are unnecessary for the purposes of this paper, we will not pursue that approach here.[6]

First, we wish to show that $\partial^2 C/\partial k^2 \leq 0$. This coupled with the result $\partial C/\partial k > 0$, obtained in Eq. (14) above, implies that commuting cost increases at a nonincreasing rate with distance from the CBD. $\partial^2 C/\partial k^2 \leq 0$ is obtained from the comparative-static result

$$\frac{\partial\phi}{\partial k} = \frac{1}{D}\begin{vmatrix} E_{ss} & V_s - \phi & 0 \\ V_s - \phi & 0 & 0 \\ -t & -s & -1 \end{vmatrix} = \frac{(F_s/t)^2}{D} \leq 0. \quad (15)$$

The marginal cost of commuting unambiguously falls with distance if $F_s \neq 0$. This result holds in the standard urban model if the total commuting cost function increases at a decreasing rate with distance (in Muth's notation, if $T_{kk} < 0$). The marginal cost of commuting is independent of distance if $F_s = 0$. This holds in the standard urban model if the total commuting cost function increases at a constant rate with distance (in Muth's notation, if $T_{kk} = 0$). In the standard urban model, these results are assumed; in our model, they are derived.

The intuition behind these results is not too difficult to see. If s were constant (implying a fixed mode, or $F_s = 0$), then $\partial s/\partial k = 0$, i.e., one would travel a greater distance at the same speed. But if speed does not change with distance, then neither does MC. (Recall from Eq. (14) that $\mathrm{MC} = [V(s) + w]/s$.) On the other hand, if s were not constant (implying mode choice, or $F_s \neq 0$) then $\partial s/\partial k \neq 0$, i.e., one could travel a greater distance at a faster or slower speed. To travel a greater distance, however,

[6] The appendix mentioned in note 5 contains these results.

one would not choose to change speed (i.e., mode) unless total commuting cost rose at a slower rate with the new mode than with the old one, i.e., unless $\partial \mathrm{MC}/\partial k < 0$. So, if $F_s \neq 0$, then $\partial \mathrm{MC}/\partial k < 0$.

Next, we obtain the wage-rate effects. The following comparative-static results are obtained when w changes:

$$\frac{\partial s}{\partial w} = \frac{1}{D}\begin{vmatrix} 0 & V_s - \phi & -t \\ -1 & 0 & -s \\ 0 & -s & 0 \end{vmatrix} = -\frac{st}{D} > 0 \tag{16}$$

$$\frac{\partial t}{\partial w} = \frac{1}{D}\begin{vmatrix} E_{ss} & 0 & -t \\ V_s - \phi & -1 & -s \\ -t & 0 & 0 \end{vmatrix} = \frac{t^2}{D} < 0 \tag{17}$$

$$\frac{\partial \phi}{\partial w} = \frac{1}{D}\begin{vmatrix} E_{ss} & V_s - \phi & 0 \\ V_s - \phi & 0 & -1 \\ -t & -s & 0 \end{vmatrix} = \frac{t(V_s - \phi) - sE_{ss}}{D} = -\frac{F_s + sE_{ss}}{D}. \tag{18}$$

These results say that if the wage rate rises, then the commuter will choose a faster mode and spend less time commuting. The results on time and speed seem intuitively plausible to us and consistent with causal observation. We shall discuss the implications of the third result thoroughly below.

2. PROPERTIES OF MC AND $\epsilon_{\mathrm{MC},w}$

A. *Relation between Marginal Commuting Cost and the Wage Rate*

From the comparative-static analysis of the cost minimization model, the variation in MC with w is given by Eq. (18) above. We shall rewrite that result in a form that is more useful for our purposes. From Eqs. (3) and (11), we can derive $D = -(2F_s + sE_{ss})s < 0$. Substituting D into Eq. (18), the variation in MC with w is given by

$$\frac{\partial \mathrm{MC}}{\partial w} = \frac{F_s + sE_{ss}}{(2F_s + sE_{ss})s}, \tag{19}$$

which implies that

$$\frac{\partial \mathrm{MC}}{\partial w} \gtreqless 0 \qquad \text{as } F_s + sE_{ss} \gtreqless 0. \tag{20}$$

[8]Since $C = E + tw$, then $\mathrm{MC} = E_k + t_k w$. Hence, if E_k and t_k are independent of w, $\partial \mathrm{MC}/\partial w = t_k > 0$.

Hence, the marginal cost of commuting may rise, fall, or remain unchanged as the wage rate changes.[7] In the typical model of urban household behavior, MC is an increasing function of the wage rate (recall Muth's assumption that $T_{ky} > 0$ for a change in the wage rate).[8] This occurs in our model if $F_s + sE_{ss} > 0$. A special case is when $F_s = 0$. Then, from Eq. (19), $\partial \mathrm{MC}/\partial w = 1/s > 0$. (An anonymous referee brought this possibility to our attention.)

Since the idea that the marginal cost of commuting may be independent of or decrease with the wage rate is unfamiliar (although not unknown; see [8]), a hypothetical example might help convince the reader of the plausibility of this possibility. Suppose a commuter has a choice between only two modes, bus and auto. Table 1 illustrates the case in which the bus, a low-cost but slower mode, has the lower total cost of a given commute than the auto, a high-cost but faster mode, for lower income individuals, while the opposite is the case for higher income individuals. (The numbers are purely hypothetical and are not meant to represent actuality.) Consequently, the auto will be chosen by the high-income individual while the bus is chosen by the low-income individual. As an individual's income rises from \$6 to \$9 an hour, he will switch from bus to car, and the marginal cost of commuting will fall from \$0.40 to \$0.35 per mile, i.e., $\partial \mathrm{MC}/\partial w < 0$. (A similar example could be constructed such that $\partial \mathrm{MC}/\partial w = 0$.)

B. *Properties of the Wage-Rate Elasticity of Marginal Commuting Cost*

In the typical urban model, the wage-rate elasticity of marginal commuting cost is assumed to lie between zero and one and either to be an increasing function of the wage rate or to be independent of the wage rate. We will show that, in a model that allows for mode choice, the wage-rate elasticity of commuting cost may be negative or greater than one and that it may be a decreasing function of the wage rate. Moreover, we will show that the assumption of a constant elasticity is not admissible in the standard model as it involves an implicit assumption of mode choice if, as

[7] Using Shephard's Lemma, it may be shown that $\partial \mathrm{MC}/\partial w = \partial t/\partial k$. This implies that if time is an inferior input in commuting, then the marginal cost of commuting is decreasing with the wage rate. This is analytically the same result derived in [6], where, in our case, k replaces output and t replaces the quantity employed of an input. (In our model, however, input inferiority depends on the curvature of the E function with respect to s, while in standard production theory it depends on the shape of the isoquant.) Time will be an inferior input if $F_s + sE_{ss} < 0$, or if $F_s < -sE_{ss}$. Since $(2F_s + sE_{ss})s > 0$, then $2F_s > -sE_{ss}$. In addition, if $F_s + sE_{ss} < 0$, then if $F_s < 0$, $2F_s + sE_{ss} < 0$, which contradicts the second-order condition that $2F_s + sE_{ss} > 0$. Therefore, $F_s > 0$. Together, these results imply that time will be an inferior input and the marginal cost of commuting will fall with the wage rate if $2F_s > -sE_{ss} > F_s > 0$. $F_s > 0$ further implies that $E_{ss} < 0$ since $s > 0$.

TABLE 1
Hypothetical Commuting Costs, Bus and Car

Item	Bus	Car
Fixed costs		
Money	\$1.00	\$2.50
Time	5 minutes	0 minutes
Variable costs/mile		
Money	\$0	\$0.05
Time	4 min./mi.	2 min./mi.
Commuting distance	5 miles	5 miles
Low-wage case (\$6/hr or \$0.10/min)		
Total cost	\$3.50	\$3.75
MC	0.40	0.25
High-wage case (\$9/hr or \$0.15/min)		
Total cost	\$4.75	\$4.25
MC	0.60	0.35

is usually assumed, money costs of commuting rise with distance commuted.

The wage-rate elasticity of marginal commuting cost may be negative. This follows easily from Eq. (20) which says that the marginal cost of commuting may be a decreasing function of the wage rate. Since $\epsilon_{\mathrm{MC},w} \equiv (w/\mathrm{MC})(\partial\mathrm{MC}/\partial w)$, then, since w and MC are both positive, $\epsilon_{\mathrm{MC},w} < 0$ if $\partial\mathrm{MC}/\partial w < 0$.

The wage-rate elasticity of marginal commuting cost may be greater than one. This elasticity can be written as

$$\epsilon_{\mathrm{MC},w} = \left(\frac{w}{V+w}\right)\left(\frac{F_s + sE_{ss}}{2F_s + sE_{ss}}\right). \tag{21}$$

This follows from Eq. (4), which requires that $\mathrm{MC} = (V+w)/s$. The wage-rate elasticity of marginal commuting cost will exceed one if $F_s < -VsE_{ss}/(2V+w) < 0$.[9]

The wage-rate elasticity of marginal commuting cost may be a decreasing function of the wage rate. The change in $\epsilon_{\mathrm{MC},w}$ with respect to w is

[9] $\epsilon_{\mathrm{MC},w} > 1$ implies that $(F_s + sE_{ss})/(2F_s + sE_{ss}) > (V+w)/w > 1$. The first inequality implies that $F_s < -VsE_{ss}/(2V+w)$. $(F_s + sE_{ss})/(2F_s + sE_{ss}) > 1$ implies that $F_s < 0$. $2F_s + sE_{ss} > 0$ by virtue of the first- and second-order conditions of the cost minimization model and $s > 0$. If $(F_s + sE_{ss})/(2F_s + sE_{ss}) > 1$, then $F_s + sE_{ss} > 0$. But if $F_s < 0$, then $sE_{ss} > 0$ and $E_{ss} > 0$ since $s > 0$. Hence, $-VsE_{ss}/(2V+w) < 0$.

given by

$$\frac{\partial\epsilon_{\mathrm{MC},w}}{\partial w} = \left(\frac{F_s + sE_{ss}}{2F_s + sE_{ss}}\right)\left(\frac{V - wV_s(\partial s/\partial w)}{(V+w)^2}\right) + \left(\frac{w}{V+w}\right)\left(\frac{\partial((F_s + sE_{ss})/(2F_s + sE_{ss}))}{\partial w}\right). \tag{22}$$

The first term on the right-hand side of this equation may be of any sign since it is equal to $s(\partial \mathrm{MC}/\partial w)$. The sign of the second term is determined by the sign of its numerator, but $\partial s/\partial w > 0$ while we have not made a sign assumption about V_s; hence, its sign is ambiguous. The first term after the plus sign is positive. It can be shown that the sign of the last term depends on the sign of the following expression, which is of ambiguous sign: $(F_s E_{ss} + sF_s E_{sss} - sF_{ss} E_{ss})(\partial s/\partial w) + (sF_s E_{sst})(\partial t/\partial w)$. Thus, the variation in the wage-rate elasticity of marginal commuting cost with the wage rate may be of any sign. Note that this result holds even when $\partial \mathrm{MC}/\partial w > 0$, in which case the first term on the right-hand side of the equation is positive.

A constant wage-rate elasticity of marginal commuting cost implies mode choice. In our model with mode choice, Eq. (22) implies that $\partial\epsilon_{\mathrm{MC},w}/\partial w$ may be zero. In our model without mode choice, the wage-rate elasticity is $\epsilon_{\mathrm{MC},w} = t_k w/(E_k + t_k w)$ and $\partial\epsilon_{\mathrm{MC},w}/\partial w = E_k t_k/(E_k + t_k w)^2$, which is positive if E_k and t_k are positive.[10] For a given mode, t_k is always positive. E_k is usually assumed to be positive, although one might argue that for walking it is zero (but even shoes wear out). Consequently, models assuming the wage-rate elasticity of marginal commuting cost is constant (and that $E_k > 0$) are implicitly assuming mode choice.

C. *Summary*

In addition to the standard results, the following new possibilities may occur in a model with mode choice: $\partial \mathrm{MC}/\partial w \leq 0$, $\epsilon_{\mathrm{MC},w} < 0$, $\epsilon_{\mathrm{MC},w} > 1$, and $\partial\epsilon_{\mathrm{MC},w}/\partial w < 0$. Also, in models without mode choice, $\partial\epsilon_{\mathrm{MC},w}/\partial w = 0$ only if $E_k = 0$; if $E_k > 0$, then such models implicitly assume mode choice.

[10] If the marginal value of time spent commuting is not equal to the wage rate, then as long as it is an increasing function of the wage rate, $\epsilon_{\mathrm{MC},w}$ will still be less than or equal to one and $\partial\epsilon_{\mathrm{MC},w}/\partial w$ will be greater than zero. A commonly used assumption is that the marginal value of time spent commuting is a positive fraction, say a, of the wage rate. Then, $\epsilon_{\mathrm{MC},w} = t_k aw/(E_k + t_k aw)$ and $\partial\epsilon_{\mathrm{MC},w}/\partial w = E_k t_k a/(E_k + t_k aw)^2$.

3. INCOME AND RESIDENTIAL LOCATION

A. *How Location Depends on* $\partial\epsilon_{\mathrm{MC},w}/\partial w$ *and* $\epsilon_{q,w}$

In most discussions of urban income−location patterns within the context of the standard urban model, the wage-rate elasticity of marginal commuting cost and of housing demand are treated as constants.[11] Under these conditions, the effect of income on location is a simple matter of comparing the magnitudes of these elasticities using Eq. (1). In some cases, the wage-rate elasticity of marginal commuting cost is permitted to vary with the wage rate (see [5, 7]). Under these conditions, if the wage-rate elasticity of marginal commuting cost rises with the wage rate and lies between zero and one and if the wage-rate elasticity of housing demand is constant and greater than one, then upper income households would live farther from the CBD than lower income households. If, on the other hand, the wage-rate elasticity of housing demand is a constant lying between zero and one, then the pattern of location by income is more complicated. (This possibility was investigated in [5].)

When mode choice is permitted, the wage-rate elasticity of marginal commuting cost may be of any sign. If $\epsilon_{\mathrm{MC},w} \leq 0$ and housing is a normal good, then higher wage households will live farther from the CBD. Suppose, however, $\epsilon_{\mathrm{MC},w} > 0$ and falls with the wage rate while $\epsilon_{q,w}$ is a positive constant. Then, at some wage rate, say w_1, $\epsilon_{\mathrm{MC},w}$ will equal $\epsilon_{q,w}$. Among households with wages less (more) than w_1, those earning the lowest (highest) wages would be located farthest out. These results are reversed when $\epsilon_{\mathrm{MC},w}$ rises with the wage rate.

The pattern of urban location by income has changed over time. In the United States, before the introduction of the horse-drawn streetcar, higher income households lived nearer the center of town than did lower income households (see [13]). With the introduction of the streetcar, this pattern began to change, and it changed radically with the introduction and widespread adoption of the automobile as a means of commuting. Patterns of location and income also differ across cities at a given time. Although, in the United States, the pattern of upper income households located in the suburbs dominates, this is not the dominant pattern observed in cities elsewhere, e.g., cities in developing countries and Latin America [9, 12, 14]. A major goal of urban economics is to explain these patterns and why they differ across time and space.

The income−location pattern will change over time or differ at a given time due to times-series variation or to cross-sectional variation in the

[11] Although we emphasize the variability of the wage-rate elasticity of marginal commuting cost, the wage-rate elasticity of housing demand is, in general, also a variable. In addition, in a utility maximization model with leisure choice, its sign is ambiguous. See, for example [4].

relative magnitudes of the two wage-rate elasticities, $\epsilon_{q,w}$ and $\epsilon_{\mathrm{MC},w}$. Our model suggests that the wage-rate elasticity of marginal commuting cost depends on the wage rate, the average speed of the chosen mode, and the money cost of commuting. Consequently, time-series or cross-sectional variation in any of these variables can change the income–location pattern through a change in $\epsilon_{\mathrm{MC},w}$. In our model, changes in the money cost of commuting are represented by shifts or pivots in the E function.

B. *Time-Series Variation in Commuting Cost*

It is possible that due to the introduction of new technology, such as the automobile, there is a shift in the E function and $\epsilon_{\mathrm{MC},w}$ falls from a value higher than $\epsilon_{q,w}$ to one lower than $\epsilon_{q,w}$. LeRoy and Sonstelie [13] argue that the change in income–location patterns observed in the United States could be explained in terms of the life cycle of a commuting mode. Before the introduction of a new mode, everyone is using the same mode. If $\epsilon_{q,w} < \epsilon_{\mathrm{MC},w}$, then upper income households will live near the CBD. With the introduction of a new and faster, but more expensive, mode, the rich find it economical to switch. If the new mode reduces $\epsilon_{\mathrm{MC},w}$ sufficiently for the rich, they will now locate farther out. As real wages rise relative to the cost of the new mode, the new mode will eventually become economical for the poor. The residential pattern then reverts to the one that prevailed before the introduction to the new mode. LeRoy and Sonstelie document this pattern of change in the United States as walking gave way to the streetcar which was supplanted by the auto. Gin and Sonstelie [8] document the same process at work due to the introduction of the streetcar in 19th century Philadelphia.

Although this explanation is consistent with our model, there is another, perhaps complementary, explanation, not yet empirically tested. The change in the pattern of location that has occurred in the United States over time may have been a result of a steady rise in real wages as well as of changes in intraurban transportation technology. Suppose that the wage-rate elasticity of marginal commuting cost falls over a range of wage rates but rises over a higher range, with the constant wage-rate elasticity of housing demand falling in between the upper and lower ranges of $\epsilon_{\mathrm{MC},w}$, as shown in Fig. 1. In an earlier period of U.S. history, when the income distribution was lower and fell within the range from 0 to w_1, upper income households would live closer to the CBD. As the income distribution shifted upward and fell within the range w_1 and w_2, the pattern changed, and higher income households lived farther from the CBD. As real income continued to grow, such that the lowest was w_2, then the rich would again live closer to the center than would the poor. Of course, a full theoretical analysis of this hypothesis would require a general equilibrium model.

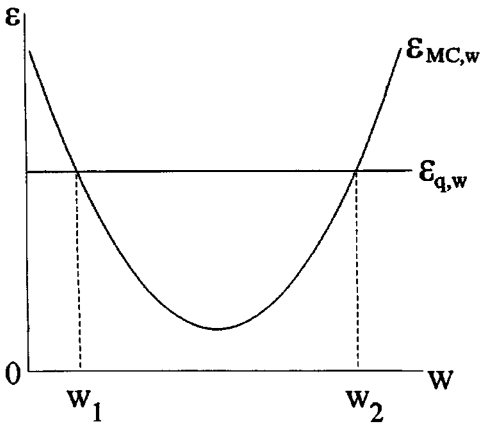

FIG. 1. Relationship between the wage-rate elasticity of marginal commuting cost and the wage-rate elasticity of housing demand as functions of the wage rate.

C. *Cross-sectional Variation in Commuting Cost*

In some cities in Latin America, the poor are found at the city edges [9, 12], while in most cities in the United States, they are found near the city center. Our model suggests that one of the reasons for such differences in income−location patterns may be due to differences in the E function which may, in turn, be the result of differences in infrastructure development and topography. Suppose the money cost of commuting, the E function, for the United States is such that $\epsilon_{\mathrm{MC},w}$ is lower than $\epsilon_{q,w}$, while the money cost function for Latin America results in an $\epsilon_{\mathrm{MC},w}$ greater than $\epsilon_{q,w}$. In this situation, there will be a tendency for lower income groups in the United States to live near the CBD, while they live near the city edges in Latin America. In developing countries low-income people can be found living at the periphery of the city, while in developed countries they are frequently located near the central part of the city [14]. If $\epsilon_{\mathrm{MC},w}$ and the wage rate are related as shown in Fig. 1, differences in real wages can be offered as an explanation for such differences in the location-by-income pattern.

4. SUMMARY AND CONCLUSIONS

Introducing mode choice into the urban residential model reveals a number of restrictions that models without mode choice place on the behavior of important determinants of residential location. In particular, we have shown that the marginal cost of commuting may be zero or negative, and that the wage-rate elasticity of marginal commuting cost can also be zero or negative. Models without mode choice restrict these variables to be positive. In addition, we find that in a model with mode choice, the wage-rate elasticity of marginal commuting cost can exceed unity, while models without mode choice restrict it to lie below unity. We show that the wage-rate elasticity of marginal commuting cost may fall with the wage rate, while models without mode choice restrict it to rise with the wage rate. Finally, the wage-rate elasticity of marginal commuting cost may be a constant in a model with mode choice but not in a model without mode choice unless the marginal money cost of commuting is zero; yet it is not uncommon to find the assumption of a constant wage-rate elasticity of marginal commuting cost in models without mode choice.

That the marginal cost of commuting and its wage-rate elasticity may behave in these ways yields insight into the relationship between income and urban residential location. The model is consistent with the theoretical development and empirical findings of LeRoy and Sonstelie [13] and Gin and Sonstelie [8], although our development is partial equilibrium while theirs employs bid−rent functions. Based on their findings and the model of this paper, it seems plausible that much of the variation in locational patterns by income that has been observed in the United States, and presumably other countries, was due to technological change in intraurban transportation. Although less well documented, it also seems plausible that differences in the money cost of commuting by alternative modes and/or in real wages explain differences in location−income patterns across countries.

Our model is partial equilibrium, so it is only suggestive of patterns of location by income when incomes vary throughout an urban area. An obvious extension, therefore, would be the development of a general equilibrium model that incorporated mode choice. We have pared down the characteristics of a mode to its average trip speed and time. In addition to these characteristics, comfort, convenience, frequency of service, and others affect the choice of mode. An extension of the model to include some other characteristics of modes might prove beneficial. Along the same lines, as one of our referees noted, a consumer might buy a car even if the bus were a less expensive commuting mode if the car were sufficiently economical for other types of trips, such as shopping and recreation trips. Extending the model in this direction might prove interesting. Although this is a theoretical paper, empirical evidence on the behavior of our E function would be desirable. Although we show that the effect of a wage-rate change on marginal commuting cost can be of any sign and that the change in the wage-rate elasticity of marginal commuting cost may rise or fall with the wage rate, empirical evidence could perhaps narrow the possibilities.

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