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MODE CHOICE, COMMUTING COST, AND URBAN HOUSEHOLD BEHAVIOR*

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ABSTRACT. In this paper, we extend the partial equilibrium urban model of DeSalvo (1985) to include mode choice. DeSalvo demonstrated that the urban model of Muth (1969) was robust to the extension to leisure choice. We show that the model is robust to mode choice as well. In addition, we derive the comparative static results that commuters choose higher speed modes for longer commutes, at higher wage rates, with greater tastes for housing, and with lower housing prices. Also, for a given distance commuted, we derive the comparative static result that commuters chose shorter duration commutes at higher wage rates. Whereas it is typically assumed that marginal commuting cost is positive and non-increasing with distance, we derive these results. Moreover, we derive the results that marginal commuting cost rises with an exogenous increase in housing price and falls with increased tastes for housing. We also explore the effects of exogenous commuting-cost changes on the endogenous variables of the model. The remaining comparative static results on housing consumption and location are qualitatively the same as in DeSalvo.

1. INTRODUCTION

Barbara Brown (1986, p. 128) began her article as follows: "Urban residential location demand theory and urban transportation mode demand theory are bodies of analysis so separate from each other as to belong primarily to different disciplines, the former to urban economics, the latter to urban transportation planning. The separate development of these two bodies of theory presents no problem if residential location and transportation mode are independent goods, because if they are, demand for each can be modeled without considering the other." She demonstrated, however, that, rather than being independent goods, residential location and mode choice were simultaneously

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determined. She concluded, therefore, that location "prices" should not be excluded from mode demand equations.

While Brown's purpose was to bring urban economic theory to bear on the theory and practice of urban transportation planning, LeRoy and Sonstelie (1983) took a different tack on the relationship between mode choice and location. They argued that mode choice and commuting cost were fundamental to understanding the spatial distribution of households by income in urban areas. By extending the standard monocentric model to include two modes, a fast but expensive one and a slow but inexpensive one, they showed that "[i]f the income elasticity of demand for housing is less than that of the marginal cost of commuting by either mode, then the rich will live on the edge of the city only if the faster mode of transportation is cheap enough that the rich opt to use it, but is costly enough that the poor do not" (p. 69). If, on the other hand, the faster mode is too costly even for the rich, or is cheap enough even for the poor, then all will use the same mode. This implies that the rich will live closer to their workplace, the Central Business District (CBD). Consequently, for LeRoy and Sonstelie, changes in the cost of commuting by various modes and the resulting mode-choice changes determine the urban spatial distribution of households by income.

Although the simultaneous determination of mode choice and residential location was recognized by others before Brown, she provided a model that introduced mode choice as a continuous variable, which had not previously been done to our knowledge.[1] Similarly, LeRoy and Sonstelie advanced a hypothesis relating mode choice to residential location that had not to our knowledge previously been presented. Based on these analyses, it seems clear to us that the interrelationship between mode choice and residential location is important to understand the forces determining residential location decisions. We attempt, therefore, to provide a theory of residential location with mode choice. Our model follows the tradition of urban partial equilibrium analysis pioneered by Alonso (1964) and Muth (1969). More specifically, our model is an extension of DeSalvo (1985), which is itself an extension of Muth.

Our approach is in the spirit of Brown but differs in many important respects. She included commuting time in the household's utility function to capture the disutility of commuting, but she did not consider other time components or a time constraint. We, instead, include leisure in the utility function and impose a total time constraint, which includes leisure, commuting, and work time. In our model, these assumptions fix the marginal value of time spent commuting at the wage rate which is independent of other exogenous variables, whereas Brown's assumptions cause the value of time spent

[1]Prior to Brown, others who recognized the relationship between mode choice and residential location include Dendrinos (1976), McCarthy (1977), Arnott and MacKinnon (1977), Kain and Fauth (1978), and Anas and Moses (1979) as well as those cited by Brown: Lerman (1977), Anas (1981, 1982), and Anas and Chu (1984).

commuting to depend on the exogenous variables of her model.[2] This difference may, in part, explain why we are able to obtain more unambiguous comparative static results. We use a two-stage approach, in which the first stage employs commuting-cost minimization subject to a distance–speed–time constraint, while the second stage uses utility maximization subject to budget and time constraints. Although she mentioned the possibility of a two-stage approach (p. 131), Brown's analysis was based on utility maximization subject to a budget constraint and a constraint relating to distance traveled, commuting time, and the time it takes a mode to travel a unit distance. In the utility-maximization models of both approaches, location is indexed by the distance traveled between residence and workplace, taken as the CBD. However, in our cost-minimization model, residential location is not an issue, and distance refers only to the distance between origin and destination. In Brown, the money cost of commuting is the cost per unit distance (which is a function of distance) times distance, whereas we have a somewhat more general form that allows the money cost of commuting to vary with both commuting time and speed. Brown assumes a mode is characterized by the time it takes to travel a unit distance, whereas we use average speed to characterize a mode. Obviously, these variables are simply the inverses of each other.

Brown's main result is to show that mode choice and location cannot be independent even with additive preferences over commuting time, housing consumption, and a composite commodity. This result, as Brown notes, is in contrast to the standard assumption of transportation economics that mode choice and location are independent. While we arrive at the same conclusion, our approaches differ. In Brown, a change in commuting distance changes the marginal utility of commuting time, which changes the comparative advantage of different modes, causing a change of mode. Thus, in her model, mode choice works through the utility function. In our model, the mode–distance relation is determined from a cost-minimization model and is therefore independent of the utility function. Instead, the relation between distance and mode choice is determined by fixed commuting cost. With changes in commuting distance, the commuter is induced to change mode because it is possible to spread fixed costs over a greater distance commuted. We conjecture that a model with commuting time in the utility function and commuting-cost minimization would provide both of these explanations for the relationship between mode choice and commuting distance.

The only comparative static results Brown presents are the effects of a change of income on commuting time, location, housing consumption, and mode choice, and these are all indeterminate. These results are obtained under the assumption of additive utility. From our cost-minimization model, we obtain the comparative static results that higher wage earners use faster

[2] Our model can accommodate any value of travel time (VOTT) as long as it is proportional to the wage rate. When VOTT is a constant fraction of the wage rate w, say αw, where $0 < \alpha < 1$, we must replace wt in our model by αwt, where t is the travel time.

modes and spend less time commuting and that people who live farther from their destination use faster modes and experience lower marginal commuting cost. We also show how mode choice, commuting time, and marginal commuting cost are affected by factors that change the fixed and variable costs of commuting. From our utility-maximization model, we find that, if housing is a normal good, households with higher nonwage incomes live farther from the CBD and that those with greater tastes for housing and facing lower housing prices consume more housing and live farther from the CBD. We also explore the effects of exogenous commuting-cost changes on housing consumption and location. These results are obtained under general assumptions about the utility function. These, and other, comparative static results of the constrained utility-maximization model are the same, where, comparable as those of DeSalvo (1985). However, in contrast to that model, mode choice is endogenous in our model. Thus, the basic urban model is robust to the leisure-choice extension, as shown by DeSalvo, and to the mode-choice extension, as shown in this paper.

As already noted, Brown defines a mode in terms of the average time spent traveling a unit distance, whereas we define a mode in terms of its average speed. This also seems to be the definition that LeRoy and Sonstelie use although they are thinking of modes as discrete entities, while in our use the mode-choice variable is continuous. Our definition is similar to the "abstract mode" concept of Quandt and Baumol (1966).

As just noted, others have used continuous variables to represent modes, but that approach may nevertheless trouble some readers. For our purposes, we believe this approach has decided advantages over the treatment of modes as discrete physical entities (but we are aware of problematical interpretations of this assumption, some of which we note later).[3] Obviously, continuity permits the use of calculus, whereas models with discrete variables are more cumbersome to analyze. In addition, using continuous modes allows us to obtain numerous theoretically unambiguous, and therefore potentially empirically refutable, qualitative effects, most of which have not to our knowledge been obtained when discrete modes are assumed.

In addition to these reasons, continuity permits considerable flexibility in the treatment of modes, an advantage not available when treating modes as discrete entities. In our model, for any given commuting distance, the set of alternative average speeds constitutes the choice set of the decision maker. This set includes many more average speeds than is implied by a small number of available discrete physical modes, such as car, bus, bike, and walking. Thus, our approach provides a way of handling historical modes no longer in use (such as the horse-drawn trolley), those not yet in existence

[3]General equilibrium models with discrete mode choice include Sasaki (1989, 1990) and Anas (1995).

(such as Star Trek's transporter), existing but economically inefficient modes (such as waterway travel), and, of course, present-day modes.

Even in the context of existing physically discrete modes, a variety of alternative average speeds are available, each being a "mode" in our model. A commuter can combine alternative discrete modes, such as park and ride, and ride and walk, thereby choosing from among a large number of alternative average speeds. In addition, when different travel routes are used, the same physical mode may generate different average speeds. Furthermore, a commuter using a given physical mode can change average speed by varying arrival and departure times to avoid congestion. Moreover, under our identification of a mode as average speed between home and work, both waiting time and linehaul time are included. Thus, two modes, such as car and bus, sharing the same infrastructure, such as a highway, may move at the same linehaul speed, such as 50 miles per hour. Despite this, these different physically discrete modes would not be the same mode in our model because of differing waiting times.

A drawback to our definition is that it would classify two traditional modes, such as high-speed rail and the car, as the same mode if they both achieved the same average commuting speed for the same trip. Nevertheless, when considering modes typically used for commuting, either now or in the past, it seems reasonable to us to array them in terms of their average speeds.[4]

2. THE MODE-CHOICE MODEL[5]

Commuting time, average commuting speed, and commuting distance are related by the equation

$$k = st \tag{1}$$

where k is commuting distance, s is average trip speed, and t is trip time.

We define the money cost of commuting as $E(s,t) = F(s) + V(s)t$.[6] The total money cost of commuting, $E(s,t)$, is a function of both speed and time and consists of a component independent of time, $F(s)$, and a component that varies with time, $V(s)t$. We refer to the former as fixed cost and the latter as variable cost. $F(s)$ may have a component that is independent of both time and speed, which is typically

[4]Jørgensen and Polak (1993) have developed a model in which speed is chosen to minimize the cost of travel, which includes the value of time, the expected cost of an accident, and the expected cost of a speeding violation. In our model, average speed is intended to differentiate modes, not to represent the best speed for a given mode.

[5]Some of what follows is contained in DeSalvo and Huq (1996). Here, we present only enough to make this paper self-contained. Also, the thrust of that article was the investigation of the marginal cost of commuting distance in the presence of mode choice and its relation to the spatial distribution of households by income, a subject not pursued here.

[6]Using the more general formulation, $E(s,t)$, yields all but two of the comparative static results we obtain with the less general form, $F(s) + V(s)t$. We view the additional results as important enough to warrant the more restrictive formulation.

called fixed cost. Consequently, $F(s)$ is fixed in the sense that once a mode is chosen, this cost component does not change with travel time, nor, therefore, with distance, e.g., auto license fees and bus fares that do not involve a transfer.

This usage has the drawback that a Hyundai and a Ferrari with the same average commuting speed would have the same fixed cost. Another drawback, one might contend, is that our fixed cost refers only to commuting trips. In choosing a mode, however, consumers consider alternative uses, such as shopping and recreation, perhaps combining these uses in multipurpose trips. To deal with this objection, we can define $F(s)$ as net of a mode's non-commuting benefits. To see this, assume there is no commuting. From the household's utility-maximization problem, one can calculate the net benefits of all non-commuting uses, including multipurpose trips, of all alternative modes s. Let NB(s) represent these net benefits. In terms of our commuting mode-choice model, NB(s) becomes a fixed benefit of non-commuting uses of different modes, and $F(s)$ may be interpreted as fixed money costs of mode s minus $NB(s)$. This implies that each mode is associated with fixed monetary costs as well as with fixed benefits of non-commuting travel. Any factor that affects benefits from non-commuting travel will have the same effect as that of a change in the F function and can be dealt with in the comparative statics as an exogenous shift in that function.

The term $V(s)t$ implies that $E(s,t)$ rises with t at a constant rate, $V(s)$, e.g., one more hour's driving adds the same amount to cost as the previous hour did. This seems to us a reasonable assumption although it is possible to think of exceptions, e.g., even at a constant speed, depreciation due to time may be higher in the second hour of driving than in the first hour.

We make no assumption about the sign of E_s.[7] It is an implication of the first-order conditions that $E_s > 0$, i.e., the money cost of commuting rises with the average speed of a mode. We assume $F \geq 0$, which allows for the possibility that some modes may have no fixed cost, e.g., walking or public transit fares that vary with distance only. We also assume $F_s > 0$, i.e., fixed cost rises with speed, e.g., a car's loan payments, license fees, and insurance costs independent of time cost more than a bus fare. It is possible to think of situations where this assumption is violated, e.g., a taxicab ride may be faster than a car ride but involves lower fixed cost, i.e., $F_s < 0$. We assume $V \geq 0$, which allows for the possibility that some modes may have no variable cost, e.g., a bus fare that does not vary with distance traveled. V_s may be of any sign except that if it is negative, its magnitude must be such that $E_s = F_s + V_s t$ is positive to be consistent with the first-order conditions.

We assume that leisure is the only time argument of the utility function and that there are no constraints on work time. Thus, the marginal opportunity cost of time is the wage rate (w) and the total opportunity cost of commuting time is wt.

[7]Throughout this paper, we denote differentiation with subscripted letters.

Total commuting cost is given by

(2) $$C = E(s,t) + wt = F(s) + V(s)t + wt$$

The mode-choice problem is to choose s and t to minimize (2) subject to (1). Setting this problem up in Lagrangian form, we have

$$\Gamma = E(s,t) + wt + \phi(k - st) = F(s) + V(s)t + wt + \phi(k - st)$$

where Γ is the Lagrangean function, ϕ is the undetermined Lagrange multiplier, and the choice variables, previously defined, are s and t.[8]

First-order conditions are

(3) $$\Gamma_s = E_s - \phi t = 0$$
$$\Gamma_t = V + w - \phi s = 0$$
$$\Gamma_\phi = k - st = 0$$

The second-order condition is

$$D = \begin{vmatrix} E_{ss} & V_s - \phi & -t \\ V_s - \phi & 0 & -s \\ -t & -s & 0 \end{vmatrix} < 0$$

where $E_{ss} \equiv F_{ss} + V_{ss}t$.

Before turning to the comparative statics of the model, we note an implication of the first-order conditions. Using the envelope theorem and the first-order conditions, we have

$$\Gamma_k \equiv C_k = \phi = (V + w)/s = E_s/t > 0$$

[8]For any given distance, it is possible to substitute (1) into (2) and optimize over s alone. That approach, however, complicates the derivation of the marginal cost of distance and the effects of exogenous variables on endogenous variables. Consequently, we use the standard Lagrangian approach, which allows us to use total differentials of the first-order optimization conditions and Cramer's rule to derive the comparative static results. For a discrete choice version of this model, let $C^i(k,w) = F^i + V^i k + wt^i k$, where the superscript indexes the mode and k is the distance traveled. Then the minimized total cost is $C^*(k,w) = \min_i[C^i(k,w)]$. To illustrate how this model would work, assume two modes, a faster mode (car), where $i = c$, and a slower mode (bus), where $i = b$, so that $t^b > t^c$. Assume further that fixed money cost of the bus is less than that of the car, so that $F^b < F^c$. Set $C^b = C^c$, and solve for k', the distance at which both modes generate the same total cost, getting $k' = (F^c - F^b)/[w(t^b - t^c) + (V^b - V^c)]$. When one mode does not dominate for all locations (i.e., for $k' > 0$), then in this model, for trips $k < k'$, bus is used, and for trips $k > k'$, car is used. When w rises, k' falls, as people use cars for shorter trips than before. If bus fixed costs rise, k' falls, and if car fixed costs rise, k' rises. Changes in variable costs work the same way but are a little more complicated. The ability to deal easily with abstract modes is lost, however, and hence we prefer the continuous model, rather than the discrete model.

Thus, total commuting cost rises with distance traveled and E_s is positive. We refer to C_k as marginal (distance) commuting cost. It is the wage-rate elasticity of marginal commuting cost that plays such an important role in the distribution of urban households by income.[9]

3. COMPARATIVE STATIC ANALYSIS OF THE COST-MINIMIZATION MODEL

In this section, we examine the effects on mode choice (s), commuting time (t), and marginal commuting cost (ϕ) of changes in commuting distance (k), the wage rate (w), and the fixed and variable costs of commuting. We note here that most of the comparative static results use the fact that $V_s - \phi = -F_s/t$, which may be obtained from the first-order condition, Equation (3), and the fact that $E_s = F_s + V_s t$.

To deal with changes in the fixed costs of commuting, we introduce a shift parameter as follows: $F = F(s,\alpha)$, where $F_\alpha \geq 0$ and $F_{s\alpha} \gtreqless 0$. To illustrate, suppose a toll to enter the CBD is imposed on all modes of transportation, such as exits in Singapore, then $F_\alpha > 0$ and $F_{s\alpha} = 0$ (see Figure 1, where the dot–dash line illustrates a parallel upward shift of the solid $F(s)$ line).[10] If auto-licensing fees rise without a change in bus fares, then $F_\alpha = 0$ at the bus's average speed and $F_{s\alpha} > 0$ (see Figure 1, where the dashed line shows the solid $F(s)$ line pivoted upward about the bus's average speed, i.e., faster modes become more expensive). Finally, suppose that the bus fare is composed of two parts, one fixed and the other proportional to distance, and that the fixed part increases, then, $F_\alpha = 0$ at the car's average speed and $F_{s\alpha} < 0$ (see Figure 1, where the dotted line shows the solid $F(s)$ line pivoted downward about the car's average speed, i.e., slower modes become more expensive). Of course, in both of these latter illustrations, it is possible that $F_\alpha > 0$ (graphically the $F(s)$ curve shifts up and pivots up or down). We refer to exogenous changes that produce $F_\alpha > 0$ and $F_{s\alpha} = 0$ as pure level effects, $F_\alpha = 0$ and $F_{s\alpha} \gtrless 0$ as pure slope effects, and $F_\alpha > 0$ and $F_{s\alpha} \gtreqless 0$ as combined level and slope effects.

For the variable cost function, we assume $V = V(s,\beta)$, $V_\beta \geq 0$, and $V_{s\beta} \gtreqless 0$. For example, a per-mile commuting toll is represented as $V_\beta > 0$ and $V_{s\beta} = 0$ (Figure 1 may be used to interpret this and other effects by simply replacing F by V); an increase in the price of gasoline that increases the cost of operating a car by more than that of riding a bus is represented as $V_\beta > 0$ and $V_{s\beta} > 0$; an increase in the bus fare per mile that increases the cost of a bus ride but leaves the cost of operating a car unaffected is

[9]Muth (1969, pp. 20, 31) originally investigated the elasticity of C_k with respect to w. (Actually, Muth used total income, not the wage rate.) DeSalvo and Huq (1996) summarized the issue and investigated the properties of marginal commuting cost and its elasticity.

[10]For simplicity, we use straight lines in Figure 1, but the functions they represent may be curved.

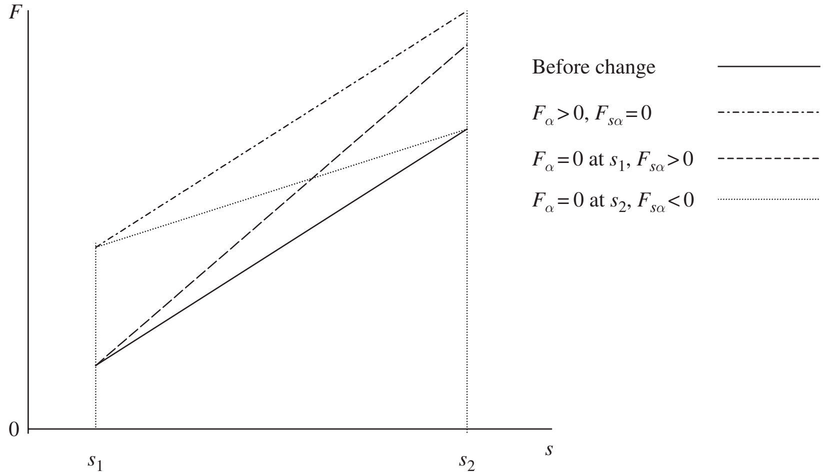

FIGURE 1: Shifts in the F Function.

represented as $V_\beta = 0$ at the car's average speed and $V_{s\beta} < 0$; and the imposition of an auto-only per-mile toll is represented as $V_\beta = 0$ at the bus's average speed and $V_{s\beta} > 0$. For the last two, it is also possible for $V_\beta > 0$.

The following equation forms the basis for the comparative static analysis

$$(4) \qquad \begin{bmatrix} E_{ss} & V_s - \phi & -t \\ V_s - \phi & 0 & -s \\ -t & -s & 0 \end{bmatrix} \begin{bmatrix} ds \\ dt \\ d\phi \end{bmatrix} = \begin{bmatrix} -F_{s\alpha} & -V_{s\beta}t & 0 & 0 \\ 0 & -V_\beta & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} d\alpha \\ d\beta \\ dw \\ dk \end{bmatrix}$$

We obtained Equation (4) by totally differentiating the first-order conditions, rearranging the resulting equations with differentials of endogenous variables on the left-hand side and differentials of exogenous variables on the right-hand side, and writing the systems of equations in matrix notation.

Commuting Distance Effects

The following comparative static results are obtained when commuting distance changes

$$\frac{\partial s}{\partial k} = \frac{1}{D} \begin{vmatrix} 0 & V_s - \phi & -t \\ 0 & 0 & -s \\ -1 & -s & 0 \end{vmatrix} = \frac{s(V_s - \phi)}{D} = -\frac{F_s(s/t)}{D} > 0$$

$$\frac{\partial t}{\partial k} = \frac{1}{D}\begin{vmatrix} E_{ss} & 0 & -t \\ V_s - \phi & 0 & -s \\ -t & -1 & 0 \end{vmatrix} = \frac{t(V_s - \phi) - sE_{ss}}{D}$$

$$= -\frac{F_s + sE_{ss}}{D} \gtrless 0 \text{ as } F_s + sE_{ss} \gtrless 0$$

$$(5) \qquad \frac{\partial \phi}{\partial k} = \frac{1}{D}\begin{vmatrix} E_{ss} & V_s - \phi & 0 \\ V_s - \phi & 0 & 0 \\ -t & -s & -1 \end{vmatrix} = \frac{(F_s/t)^2}{D} < 0^{11}$$

Thus, faster modes are chosen for longer commutes, but the qualitative effect of distance on commuting time is ambiguous.[12] The standard urban model with commuting time assumes that commuting time rises at a non-increasing rate with distance traveled. Our model, on the other hand, allows for the intuitively plausible case that a commuter employing a faster mode but traveling a greater distance might take less time to reach the CBD.

In the standard urban model without mode choice, starting with Muth (1969), total commuting cost is assumed to rise at a non-increasing rate with distance, and hence marginal commuting cost is independent of or falls with distance. In our model, however, total commuting cost rises at a decreasing rate with distance and would rise at a constant rate only if $F_s = 0$, i.e., if fixed commuting cost does not increase with faster modes. Lower marginal commuting cost is associated with faster modes for the following reason. Once a mode is chosen, marginal commuting cost is independent of distance because $F_s = 0$. Consequently, the total commuting-cost equation can be written as $C = F + C_k k$, where C_k stands for marginal commuting cost. For any given distance, $C' = F' + C'_k k > F + C_k k = C$, where primes indicate costs associated with some non-chosen mode. Thus $F > F'$ implies $C_k < C'_k$. The assumption that faster modes are associated with higher fixed cost ($F_s > 0$) implies that faster modes must be associated with lower marginal commuting cost.

[11]The results for s and ϕ are ambiguous if the more general money commuting cost function, $E(s,t)$, is used instead of the more restrictive one, $F(s) + V(s)t$.

[12]An anonymous reviewer stated that whether or not a transit mode would be attractive for commuting would depend on the home and work locations relative to the transit network's (main) nodes, while the location of such nodes would not vary continuously by trip length as we implicitly assume. This observation implicitly assumes that the residential location is given and that infrastructure may affect mode choice. In terms of our model, however, the choice of living near a "transit network node" (e.g., living close to a subway station) and "the use of that transit system" (e.g., commuting by subway) are simultaneously determined.

Wage-Rate Effects

The following comparative static results are obtained when w changes

$$\frac{\partial s}{\partial w} = \frac{1}{D}\begin{vmatrix} 0 & V_s - \phi & -t \\ -1 & 0 & -s \\ 0 & -s & 0 \end{vmatrix} = -\frac{st}{D} > 0$$

$$\frac{\partial t}{\partial w} = \frac{1}{D}\begin{vmatrix} E_{ss} & 0 & -t \\ V_s - \phi & -1 & -s \\ -t & 0 & 0 \end{vmatrix} = \frac{t^2}{D} < 0$$

$$\frac{\partial \phi}{\partial w} = \frac{1}{D}\begin{vmatrix} E_{ss} & V_s - \phi & 0 \\ V_s - \phi & 0 & -1 \\ -t & -s & 0 \end{vmatrix} = \frac{t(V_s - \phi) - sE_{ss}}{D}$$

$$= -\frac{F_s + sE_{ss}}{D} \gtrless 0 \text{ as } F_s + sE_{ss} \gtrless 0$$

A commuter will unambiguously choose a faster mode and spend less time commuting at a higher wage. The effect of the wage rate on marginal commuting cost is, however, ambiguous, depending on the sign of $F_s + sE_{ss}$. If, for example, $F_s + sE_{ss} > 0$, then marginal commuting cost rises with w, which is the case in the standard model without mode choice (again, starting with Muth, who used income instead of the wage rate). In this model, however, marginal commuting cost can fall with the wage rate, which allows the result that LeRoy and Sonstelie discuss. In other words, higher incomes may cause people to adopt modes that have lower marginal commuting costs.

Fixed Cost Effects

The following comparative static results are obtained when the fixed money cost of commuting changes

$$\frac{\partial s}{\partial \alpha} = \frac{1}{D}\begin{vmatrix} -F_{s\alpha} & V_s - \phi & -t \\ 0 & 0 & -s \\ 0 & -s & 0 \end{vmatrix} = \frac{s^2 F_{s\alpha}}{D} \lessgtr 0 \text{ as } F_{s\alpha} \gtrless 0$$

$$\frac{\partial t}{\partial \alpha} = \frac{1}{D}\begin{vmatrix} E_{ss} & -F_{s\alpha} & -t \\ V_s - \phi & 0 & -s \\ -t & 0 & 0 \end{vmatrix} = -\frac{stF_{s\alpha}}{D} \gtrless 0 \text{ as } F_{s\alpha} \gtrless 0$$

$$\frac{\partial \phi}{\partial \alpha} = \frac{1}{D}\begin{vmatrix} E_{ss} & V_s - \phi & -F_{s\alpha} \\ V_s - \phi & 0 & 0 \\ -t & -s & 0 \end{vmatrix} = -\frac{F_s(s/t)F_{s\alpha}}{D} \gtrless 0 \text{ as } F_{s\alpha} \gtrless 0$$

All of these results depend on the sign of $F_{s\alpha}$, and none depends on F_{α}, i.e., there is no level effect, only a slope effect. Obviously, there are three possibilities, which can occur under a variety of circumstances.

Consider a change in an exogenous variable that increased fixed cost by an equal amount for all modes (i.e., $F_{s\alpha} = 0$), for example, a lump-sum tax on all modes. In such a case, there would be no incentive to change the commuting mode; hence commuting time and marginal commuting cost would be unaffected.

Consider a change in an exogenous variable that increased fixed cost by more for a faster mode (i.e., $F_{s\alpha} > 0$), for example, an increase in an automobile license fee. Such a change removes some of the cost advantage of the faster mode. Hence the commuter chooses a slower mode, and commuting time and marginal commuting cost increase. Since the slower mode (with the lower fixed cost) was not chosen in the first place, its marginal commuting cost must have been higher than that of the faster mode (with the higher fixed cost). Thus, the slower mode is associated with the higher marginal commuting cost.

Finally, consider a change in an exogenous variable that makes the faster mode relatively cheaper (i.e., $F_{s\alpha} < 0$), for example, an increase in the fixed component of a bus fare. This leads to choice of a faster mode, lower commuting time, and a decrease in marginal commuting cost. The explanation for this result is the reverse of that of the previous case.

Variable Cost Effects

The following comparative static results are obtained when the variable money cost of commuting changes

$$\frac{\partial s}{\partial \beta} = \frac{1}{D}\begin{vmatrix} -V_{s\beta}t & V_s - \phi & -t \\ -V_{\beta} & 0 & -s \\ 0 & -s & 0 \end{vmatrix} = -\frac{stV_{\beta}}{D} + \frac{s^2tV_{s\beta}}{D} \gtrless 0 \text{ as } V_{\beta} \gtrless sV_{s\beta}$$

$$\frac{\partial t}{\partial \beta} = \frac{1}{D}\begin{vmatrix} E_{ss} & -V_{s\beta}t & -t \\ V_s - \phi & -V_{\beta} & -s \\ -t & 0 & 0 \end{vmatrix} = \frac{t^2V_{\beta}}{D} - \frac{st^2V_{s\beta}}{D} \lessgtr 0 \text{ as } V_{\beta} \gtrless sV_{s\beta}$$

$$\frac{\partial \phi}{\partial \beta} = \frac{1}{D}\begin{vmatrix} E_{ss} & V_s - \phi & -V_{s\beta}t \\ V_s - \phi & 0 & -V_{\beta} \\ -t & -s & 0 \end{vmatrix} = \frac{(V_s - \phi)tV_{\beta} + st(V_s - \phi)V_{s\beta} - E_{ss}sV_{\beta}}{D}$$

$$= -\frac{(F_s + sE_{ss})V_{\beta}}{D} - \frac{sF_sV_{s\beta}}{D} \gtrless 0 \text{ as } (F_s + sE_{ss})V_{\beta} \gtrless sF_sV_{s\beta}$$

In these equations, we have split the comparative static result into two parts to emphasize the role played by the level and slope effects of changes in exogenous variables on the variable cost function. In general, to know how exogenous changes affect speed and time, we must know the sign of the slope effect, $V_{s\beta}$. In addition, we need to know the sign of $F_s + sE_{ss}$ to know how marginal commuting cost reacts to exogenous changes. We get some interesting results nevertheless.

We could have exogenous changes that affect the level of variable cost but not the rate at which it changes with speed, i.e., $V_\beta > 0$ and $V_{s\beta} = 0$ (a pure level effect). In this case, the speed and time results are unambiguous whereas the marginal cost result is ambiguous. For example, an increase in a per-mile commuting toll causes commuters to choose a faster mode and spend less time commuting. It is interesting to note that if $V_\beta = 1$ and $V_{s\beta} = 0$, then the comparative static results are the same as those of a wage change (in the matrix on the right-hand side of Equation (4), column 2 equals column 3). This is not difficult to understand when one realizes that in this model the wage rate is the cost of time. Thus, as the cost of time rises, commuters choose faster modes and reduced commuting time. The effect on marginal commuting cost is ambiguous.

On the other hand, we could have changes that affect the rate at which variable cost changes with speed but not the level of variable cost, i.e., $V_\beta = 0$ and $V_{s\beta} \gtreqless 0$ (a pure slope effect). For example imposing or increasing an auto-only per-mile toll would cause commuters to choose a slower mode, spend more time commuting, and face lower marginal commuting costs.

Finally, we could have exogenous changes that affect both the level of variable cost and the rate at which it changes with speed, i.e., $V_\beta > 0$ and $V_{s\beta} > 0$ or $V_{s\beta} < 0$ (a combined level and slope effect). For example, an increase in a per-mile toll on all commuters and a decrease in gasoline prices that affects cars more than buses will cause commuters to choose faster modes and spend less time commuting (and vice versa for an increase in gasoline prices). The effect on marginal commuting cost is, however, ambiguous.

4. THE UTILITY-MAXIMIZATION MODEL[13]

Let $s = s(w,k)$ and $t = t(w,k)$ be the speed and time functions obtained by solving the first-order conditions of the cost-minimization model (equivalent to constrained input demand functions). Then total commuting cost, including both money and time costs, is given by $C = E[s(w,k),t(w,k)] + wt(w,k) = E(w,k) + wt(w,k) = C(w,k)$. This function will be used in the constrained utility-maximization model instead of adding the $k = st$ constraint to that model. $C(w,k)$ implicitly assumes a fixed number of identical trips per unit of

[13]We could have combined cost-minimization and utility-maximization into one integrated model. That approach, however, would have made it difficult to isolate the direct effect of changes in exogenous variables on mode choice from their indirect effect through change in residential location. Since the purpose of this paper was to integrate mode choice into the residential choice model, we chose to follow a two-stage optimization procedure, in which the first stage shows the effects on mode choice without change in residential location.

time. As noted originally by Muth (1969), a variable number of trips could be introduced, but since we are not interested in explaining the determinants of the number of trips, this extension is omitted.

Utility, u, is given by the ordinal utility function, $u = u(x,q,L)$, where x is a composite commodity whose price is normalized to unity, q is the flow of housing service from a dwelling unit and the land on which it is built (and is frequently indexed by square feet of floor space), and L is leisure time. The utility function is assumed to possess continuous first- and second-order partial derivatives, and marginal utilities are assumed positive, i.e., $u_x > 0$, $u_q > 0$, and $u_L > 0$.

The household is constrained by a budget constraint, $wW + y_{nw} = x + p(k)q + E(w,k)$, and a time constraint, $M = W + L + t(w,k)$, where W is working time, y_{nw} is nonwage income, k is residential location, measured as distance from the workplace, $p(k)$ is the price per unit of housing service which depends on distance from the workplace, M is total time, and the remaining variables are as defined for the cost-minimization model. As in the standard partial equilibrium urban model, $p(k)$ is taken as parametric to the household and falls at a decreasing numerical rate with distance from the workplace, i.e., $p_k < 0$ and $p_{kk} > 0$. The workplace is usually taken to be the CBD, but it may be any center of employment for which there is sufficient demand for accessibility that housing prices rise at it is approached.[14]

The problem of the household is to maximize utility subject to the budget and time constraints. Upon combining the time and budget constraints, the problem is to maximize

$$\Lambda = u(x,q,L) + \lambda[(M - L)w + y_{nw} - x - p(k)q - C(w,k)] \tag{6}$$

where Λ is the Lagrangian function, λ is the undetermined Lagrange multiplier, and the choice variables, previously defined, are x, q, L, and k. Note that the term wt is included in the C function.

The resulting model is identical to that of DeSalvo (1985), except for our treatment of the commuting-cost function. In DeSalvo, the money cost of commuting, our $E(w,k)$, is given by the function $T(k)$, where it is assumed that $T_k > 0$ and $T_{kk} \leq 0$. This is consistent with much of the literature since Muth (1969). In DeSalvo, the commuting time function, our $t(w,k)$, is given by $C(k)$, where it is assumed that $C_k > 0$ and $C_{kk} \leq 0$. Although models with explicit time variables and constraints are less common in the urban

[14]For this reason, we do not regard our partial equilibrium model as monocentric, reserving that term for general equilibrium models with one center of employment. We believe our comparative static results should apply in the vicinity of any employment center in a polycentric urban area. General equilibrium models have become more common in urban economics since Wheaton (1974), and, while the monocentric model is still used, polycentric models are becoming more common although there is no standard model. Partial equilibrium models have always preceded general equilibrium models, and we hope that our model will lead to incorporation of our approach to mode choice in urban general equilibrium models.

literature, these assumptions are consistent with those of such models, e.g., Yamada (1972, p. 126) assumed $C_k > 0$ and $C_{kk} = 0$ and Evans (1973, pp. 32–35) assumed $C_k > 0$ and $C_{kk} < 0$. Here, the difference is that these time and money cost functions are derived from a cost-minimization problem and their properties are determined from the comparative statics of that problem. In DeSalvo and others, however, these functions are defined and their properties are assumed.

The first-order conditions for this problem are

$$\Lambda_x = u_x - \lambda = 0 \tag{7}$$

$$\Lambda_q = u_q - \lambda p = 0$$

$$\Lambda_L = u_L - \lambda w = 0$$

$$\Lambda_k = -\lambda(p_k q + C_k) = 0 \tag{8}$$

$$\Lambda_\lambda = (M - L)w + y_{nw} - x - pq - C = 0$$

Second-order conditions are

$$H = \begin{vmatrix} u_{xx} & u_{xq} & u_{xL} & 0 & -1 \\ u_{qx} & u_{qq} & u_{qL} & -\lambda p_k & -p \\ u_{Lx} & u_{Lq} & u_{LL} & 0 & -w \\ 0 & -\lambda p_k & 0 & -\lambda(p_{kk}q + C_{kk}) & 0 \\ -1 & -p & -w & 0 & 0 \end{vmatrix} > 0 \begin{vmatrix} u_{xx} & u_{xq} & u_{xL} & -1 \\ u_{qx} & u_{qq} & u_{qL} & -p \\ u_{Lx} & u_{Lq} & u_{LL} & -w \\ -1 & -p & -w & 0 \end{vmatrix} < 0$$

$$\text{and} \begin{vmatrix} u_{xx} & u_{xq} & -1 \\ u_{qx} & u_{qq} & -p \\ -1 & -p & 0 \end{vmatrix} > 0$$

The off-diagonal zeroes in H are due to the exclusion of k from the utility function, which implies $u_{xk} = u_{qk} = u_{Lk} = 0$, and to the fourth first-order condition, Equation (8), which implies $p_k q + C_k = 0$ since $\lambda = u_x > 0$ from Equation (7) and the assumption of positive marginal utilities. Note that C_k is marginal commuting cost, which was analyzed in the cost-minimization model.

Before proceeding to the comparative static analysis, we note that this model produces the familiar result, first derived by Muth (1969), that at an equilibrium the household must be at a residential location, k, such that housing price is falling at a decreasing numerical rate. This follows from Equations (7) and (8). Since $\lambda = u_x > 0$, $p_k = -C_k/q < 0$, where $C_k > 0$ from the cost-minimization model. It is shown in the Appendix that $p_{kk} > 0$. These results hold at a local optimum; if they hold globally, then they describe the properties of the housing price function. In general equilibrium urban models, these results follow from the decline in land rent with distance from the CBD and the ability to substitute land for housing structure as land rent falls. These theoretical results, both for housing price and land rent, have empirical support (e.g., Alonso, 1964, p. 172; Ball, 1973; Coulson, 1991; Evans, 1973, Ch. 5; Muth, 1969, pp. 192, 237; Wieand, 1973).

5. COMPARATIVE STATIC ANALYSIS OF THE UTILITY-MAXIMIZATION MODEL

The comparative static results of the current model that do not involve the commuting-cost function are formally identical to those of DeSalvo (1985). The reason for this is that the first- and second-order conditions of both models are identical except for terms involving commuting cost. The terms involving commuting cost are different in the two models because money and time costs of commuting are only functions of distance in DeSalvo, whereas they are functions of both distance and the wage rate in the current model. A detailed exposition of these points follows.

In DeSalvo, the first-order condition obtained by partially differentiating the Lagrangian with respect to k is $-\lambda[p_k(k)q + T_k(k) + C_k(k)w] = 0$ or, rewritten in the notation of the present paper, $-\lambda[p_k(k)q + E_k(k) + t_k(k)w] = 0$. In the current model, the comparable first-order condition is $-\lambda[p_k(k)q + C_k(k,w)] = 0$, or, upon splitting marginal commuting cost into money and time components, $-\lambda[p_k(k)q + E_k(k,w) + t_k(k,w)w] = 0$. Similarly, in the earlier model, the first-order condition obtained by differentiating the Lagrangian with respect to λ is $[M - L - C(k)]w + y_{nw} - [x + p(k)q + T(k)] = 0$, or, in the notation of this paper, $[M - L - t(k)]w + y_{nw} - [x + p(k)q + E(k)] = 0$. In the present model, it is $(M - L)w + y_{nw} - x - pq - C = 0$, or, splitting money and time costs, $[M - L - t(k,w)]w + y_{nw} - [x + p(k)q + E(k,w)] = 0$.

In addition, the second-order conditions are identical in both models except for the 4-4 element of the H determinant. In the earlier model, that element is given by $-\lambda[p_{kk}(k)q + T_{kk}(k) + C_{kk}(k)w]$, or, in the notation of this paper, $-\lambda[p_{kk}(k)q + E_{kk}(k) + t_{kk}(k)w]$. In the present paper, that element is $-\lambda[p_{kk}(k)q + C_{kk}(k,w)]$, which, upon splitting out money and time costs, becomes $-\lambda[p_{kk}(k) + E_{kk}(k,w) + t_{kk}(k,w)w]$.

In light of the preceding discussion, comparative static results due to exogenous changes in household preferences, housing price, and nonwage income are identical in both models. In presenting the mathematical formulation of the comparative static results, therefore, we suppress shift parameters in the utility and housing price functions. We present nonwage income effects because the results are used elsewhere in this paper. Finally, we introduce shift parameters into the commuting-cost function as follows: $C = C(k,w,\alpha,\beta)$, where α and β are the same as in the cost-minimization model.

The following equation forms the basis for the comparative static results. It was obtained by totally differentiating the first-order conditions, with the new commuting-cost function, rearranging terms so that the differentials of endogenous variables are on the left-hand side and differentials of exogenous variables are on the right-hand side, and writing the system of equations in matrix notation.

$$
\begin{bmatrix} u_{xx} & u_{xq} & u_{xL} & 0 & -1 \\ u_{qx} & u_{qq} & u_{qL} & -\lambda p_k & -p \\ u_{Lx} & u_{Lq} & u_{LL} & 0 & -w \\ 0 & -\lambda p_k & 0 & -\lambda(p_{kk}q + C_{kk}) & 0 \\ -1 & -p & -w & 0 & 0 \end{bmatrix} \begin{bmatrix} dx \\ dq \\ dL \\ dk \\ d\lambda \end{bmatrix} \tag{9}
$$
$$
= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ \lambda C_{k\alpha} & \lambda C_{k\beta} & \lambda C_{kw} & 0 \\ C_\alpha & C_\beta & -(M - L - C_w) & -1 \end{bmatrix} \begin{bmatrix} d\alpha \\ d\beta \\ dw \\ dy_{nw} \end{bmatrix}
$$

In the earlier model and in this one, the qualitative effects on the composite good, x, and leisure, L, are all ambiguous without stronger assumptions. Hence, we concentrate on the effects of exogenous changes on housing, q, and residential location measured as distance from the CBD, k.

Housing Preference and Price Effects

In light of considerations discussed in the introduction to this section, changes in taste for housing and housing price yield identical comparative static results in both models, and hence the details of these results are omitted and only a summary is presented.[15] Let θ_0 be an exogenous taste parameter whose increase represents a stronger taste for housing.[16] Then in both models, we have $\partial q/\partial\theta_0 > 0$ and $\partial k/\partial\theta_0 > 0$, i.e., an increase in the household's taste for housing yields the comparative static result that the household increases its housing consumption and moves farther from the CBD. Let θ_1 be an exogenous variable that shifts and/or pivots the housing price function upwards. Then, in both models, $\partial q/\partial\theta_1 < 0$ and $\partial k/\partial\theta_1 < 0$, i.e., the comparative static result that an increase in the level or slope of the housing price function decreases the household's consumption of housing and moves its residential location closer to the CBD. It should be noted that if the housing price function changes its level and slope in opposite directions, then the comparative static results are ambiguous.

In addition to these results which are common to both models, from our cost-minimization model, we may obtain results on speed, commuting time, and marginal commuting cost due to exogenous changes in tastes for

[15]The mathematics of these results is not contained in DeSalvo (1985), only discussed. They may, however, be found in DeSalvo (1986).

[16]Specifically, an increased taste for housing occurs when the marginal rate of substitution of housing for the composite commodity increases as the shift parameter increases, or, mathematically, when $\partial(u_q/u_x)/\partial\theta_0 > 0$. Additional assumptions are required for the comparative static result, which are not used elsewhere in the comparative statics. See DeSalvo (1985, p. 165).

housing and housing price indirectly through their effects on commuting distance. Exogenous changes in tastes for housing produce the following effects

$$\frac{\partial s}{\partial \theta_0} = \frac{\partial s}{\partial k}\frac{\partial k}{\partial \theta_0} > 0 \quad \frac{\partial t}{\partial \theta_0} = \frac{\partial t}{\partial k}\frac{\partial k}{\partial \theta_0} \gtrless 0 \quad \frac{\partial \phi}{\partial \theta_0} = \frac{\partial \phi}{\partial k}\frac{\partial k}{\partial \theta_0} < 0$$

Thus, an increase in tastes for housing theoretically induces the household to select a faster mode which lowers marginal commuting cost. The effect on commuting time is ambiguous because although the household chooses a faster mode, it also chooses to live farther from the CBD.

Exogenous changes in housing price produce the following effects

$$\frac{\partial s}{\partial \theta_1} = \frac{\partial s}{\partial k}\frac{\partial k}{\partial \theta_1} < 0 \quad \frac{\partial t}{\partial \theta_1} = \frac{\partial t}{\partial k}\frac{\partial k}{\partial \theta_1} \gtrless 0 \quad \frac{\partial \phi}{\partial \theta_1} = \frac{\partial \phi}{\partial k}\frac{\partial k}{\partial \theta_1} > 0$$

Thus, an increase in housing price (level and/or slope) theoretically induces the household to choose a slower mode which raises marginal commuting cost. Again, the effect on commuting time is ambiguous because the choice of a slower mode is coupled with the choice of a residential location closer to the CBD.

Nonwage Income Effects

Assuming housing a normal good (i.e., $\partial q/\partial y_{nw} > 0$), a change in nonwage income produces the following effects on housing and location

$$\frac{\partial q}{\partial y_{nw}} = \frac{H_{52}}{H} > 0 \text{ and } \frac{\partial k}{\partial y_{nw}} = \frac{H_{54}}{H} > 0$$

where H_{ij} is the minor of the ijth element of H. As nonwage income rises, the model says households will consume more housing (by assumption) and live farther from the CBD. The assumption that housing is a normal good and the second-order condition that $H > 0$ together imply that $H_{52} > 0$, which appears in all the comparative results involving q. The Appendix proves that $H_{54} > 0$, which appears in all of the comparative static results involving k.

It is interesting to note that if additive utility is assumed, i.e., the cross derivatives of the utility function are zero (i.e., $u_{xq} = u_{xL} = u_{Lq} = 0$), then $\partial q/\partial y_{nw} > 0$. This result occurs because, under these assumptions, $H_{52} = \lambda(p_{kk}q + C_{kk})pu_{xx}u_{LL} > 0$ if $u_{xx} < 0$ and $u_{LL} < 0$ (see the Appendix for a proof). Under the same assumptions, Brown was unable to sign $\partial q/\partial y$, where y is income. Brown did not distinguish wage from nonwage income or employ a time constraint, which may explain her inability to sign $\partial q/\partial y$.

We can also determine nonwage effects on commuting speed and time as well as on marginal commuting cost indirectly through their effects on commuting distance. Specifically

$$\frac{\partial s}{\partial y_{nw}} = \frac{\partial s}{\partial k}\frac{\partial k}{\partial y_{nw}} > 0 \quad \frac{\partial t}{\partial y_{nw}} = \frac{\partial t}{\partial k}\frac{\partial k}{\partial y_{nw}} \gtreqless 0 \quad \frac{\partial \phi}{\partial y_{nw}} = \frac{\partial \phi}{\partial k}\frac{\partial k}{\partial y_{nw}} > 0$$

Consequently, as nonwage income rises, the model says households choose a faster mode which raises their marginal commuting cost. The effect on commuting time is ambiguous because faster modes are associated with greater commuting distance.

Commuting-Cost Effects

The total commuting-cost function, including shift parameters, is given by $C(s,t,\alpha,\beta) = F(s,\alpha) + V(s,\beta)t + wt$. Hence, $C_\alpha = F_\alpha$, $C_\beta = V_\beta t$, and $C_w = t > 0$. We assume $C_\alpha \geq 0$, $C_\beta \geq 0$. Also, from the comparative statics of the cost-minimization model, we have

$$C_{k\alpha} = \partial\phi/\partial\alpha = [F_s(s/t)F_{s\alpha}]/D \gtreqless 0$$

$$C_{k\beta} = \partial\phi/\partial\beta = -[(F_s + sE_{ss})V_\beta + sF_sV_{s\beta}]/D \gtreqless 0$$

$$C_{kw} = \partial\phi/\partial w = t_k = C_{wk} = -(F_s + sE_{ss})/D \gtreqless 0$$

All the above appear in the comparative static results of changes in commuting cost.

We simplify the presentation of commuting-cost effects by setting $\alpha = \beta = \gamma$. We can do this because the variable commuting-cost effects are perfectly symmetric with the fixed commuting-cost effects. This can be seen by examining Equation (9). In the matrix on the right-hand side of that equation, the 4–1 and 4–2 elements are identical except for the shift parameters, as are the 5–1 and 5–2 elements.

A change in exogenous variables that affect commuting cost produces the following comparative static results on housing consumption and residential location ($H_{44} < 0$ by the second-order conditions; $H_{42} < 0$, as shown in the Appendix)

$$\frac{\partial q}{\partial \gamma} = -\frac{C_\gamma H_{52}}{H} + \frac{\lambda C_{k\gamma} H_{42}}{H} \begin{cases} < 0 \text{ if } C_{k\gamma} \geq 0 \\ > 0 \text{ if } C_\gamma = 0 \text{ and } C_{k\gamma} < 0 \\ \gtreqless 0 \text{ if } C_\gamma > 0 \text{ and } C_{k\gamma} < 0 \end{cases}$$

$$\frac{\partial k}{\partial \gamma} = -\frac{C_\gamma H_{54}}{H} + \frac{\lambda C_{k\gamma} H_{44}}{H} \begin{cases} < 0 \text{ if } C_{k\gamma} \geq 0 \\ > 0 \text{ if } C_\gamma = 0 \text{ and } C_{k\gamma} < 0 \\ \gtreqless 0 \text{ if } C_\gamma > 0 \text{ and } C_{k\gamma} < 0 \end{cases}$$

Any exogenous increase in total commuting cost affects the choice variables in two ways. The first is through a change in real income. Under the assumption that housing is a normal good, the sign of this effect is negative. The second effect works through the change in marginal commuting cost. This effect reinforces the first when marginal commuting cost increases. When marginal commuting cost is unaffected, the real-income effect determines the sign, which will be negative. Thus, any factor that leads to a higher total commuting cost as well as a higher or unchanged marginal commuting cost will unambiguously decrease housing consumption and residential distance from the CBD in our model. On the other hand, if total commuting cost increases while marginal commuting cost decreases, the two effects conflict. The increase in total commuting cost leads to a decrease in housing and location, but the decrease in marginal commuting cost gives rise to an incentive to move farther out and consume more housing. Thus, the net effect depends on which of these two dominates. Clearly the marginal-cost effect dominates if total commuting cost remains unchanged. (Note that changes in commuting cost directly affect housing consumption and location, rather than indirectly through their effect on location as is the case for the other exogenous variables.)

Wage-Rate Effects

Varying the wage rate produces the following effects

$$\frac{\partial q}{\partial w} = \frac{-\lambda H_{32} + \lambda C_{kw} H_{42} + (M - L - C_w) H_{52}}{H}$$

$$\frac{\partial k}{\partial w} = \frac{-\lambda H_{34} + \lambda C_{kw} H_{44} + (M - L - C_w) H_{54}}{H} \tag{10}$$

As is the case with the standard urban model, these results are ambiguous without stronger assumptions (but even additive utility does not remove the ambiguity). We show that these results produce the standard result that the effect of the wage rate on location depends on the relative magnitudes of the wage-rate elasticities of marginal commuting cost and of the demand for housing.

It can be shown that the following relationship holds for $i = 1, 2, 3$, and 5

$$H_{i4} = -\left(\frac{p_k}{p_{kk} q + C_{kk}}\right) H_{i2} \tag{11}$$

Expand H along the fourth row and rearrange terms to get

$$H_{44} = -\frac{H_{42} p_k}{p_{kk} q + C_{kk}} - \frac{H}{\lambda (p_{kk} q + C_{kk})} \tag{12}$$

Substituting Equations (11) and (12) into Equation (10) yields

$$\frac{\partial k}{\partial w} = -\frac{p_k}{p_{kk} q + C_{kk}} \left(\frac{-\lambda H_{32} + \lambda C_{kw} H_{42} + (M - L - C_w) H_{52}}{H}\right) - \frac{C_{kw}}{p_{kk} q + C_{kk}} \tag{13}$$

The first term on the right-hand side in brackets in Equation (13) is $\partial q/\partial w$. Hence

$$\frac{\partial k}{\partial w} \gtrless 0 \text{ as } -p_k \frac{\partial q}{\partial w} \gtrless C_{kw} \tag{14}$$

since $p_{kk}q + C_{kk} > 0$.

The preceding expression may be converted into elasticity form by multiplying the second inequality in Equation (14) by $w/(-p_k q)$, getting

$$\frac{\partial k}{\partial w} \gtrless 0 \text{ as } \frac{w}{q}\frac{\partial q}{\partial w} \gtrless \frac{wC_{kw}}{-p_k q}$$

but, $-p_k q = C_k$ from the first-order condition, Equation (8), so

$$\frac{\partial k}{\partial w} \gtrless 0 \text{ as } \varepsilon_{q,w} \gtrless \varepsilon_{C_k,w}$$

where $\varepsilon_{q,w}$ is the wage-rate elasticity of housing demand and $\varepsilon_{C_k,w}$ is the wage-rate elasticity of marginal commuting cost.

6. CONCLUSION

We have extended the urban model with leisure choice (DeSalvo, 1985) by introducing mode choice. As a consequence, we show that all of the comparative static results of the earlier model carry over to the current model. This is further testament to the robustness of Muth's original insight. Additional results are obtained, mainly having to do with mode choice, commuting distance, and marginal commuting cost. Specifically, our cost-minimization model produces the results that, ceteris paribus, higher wage earners use faster modes and spend less time commuting and that people who live farther from their destination use faster modes and experience lower marginal commuting cost. The model also shows how mode choice, commuting time, and marginal commuting cost are affected by factors that change the fixed and variable costs of commuting. Our utility-maximization model produces the results that if housing is a normal good, ceteris paribus, households with higher nonwage incomes live farther from the CBD and that those with greater tastes for housing and facing lower housing prices consume more housing and live farther from the CBD. We also explore the effects of exogenous commuting-cost changes on housing consumption and location. In contrast to most previous models, mode choice is endogenous in our model. We think these results are interesting in their own right, and they buttress the theoretical findings of Brown and the empirical findings of LeRoy and Sonstelie concerning the relationship between mode choice and commuting distance.

Our approach follows traditional urban residential location theory, in which utility is deterministic. Moreover, we index modes and mode choice with a continuous variable, average speed. Urban transportation models, by both economists and transportation engineers, starting with the work of

Domencich and McFadden (1974), usually treat mode choice in the context of a stochastic utility function and mode choice is made from among discrete modes. Anas (1982) takes the transportation economics and transportation engineering approach of discrete and stochastic mode choice as the reference point and integrates urban residential location theory into that framework. Our approach takes deterministic urban residential location theory as the reference point and integrates deterministic mode choice into that framework. In that sense, our model complements the synthesis of Anas.

The analysis of this paper shares the feature with other models of the same type that small changes can produce large effects. For example, a penny decline in bus fares ($F_{s\alpha} > 0$ and $C_{k\gamma} > 0$) may theoretically induce some commuters to shift from autos to bus transport, endure longer duration commutes, move closer to the CBD, and buy less housing. This is usually justified by saying that we are looking at long-run effects or are comparing people in different urban areas who are identical in all respects except for bus fares. We suspect that in a model where commuting modes were discrete entities, such changes would have to be of larger magnitudes to produce noticeable effects. Nevertheless, as far as we know, such results have not been obtained with models treating modes as discrete. Thus, the assumption of continuity permits us to derive many interesting results that are apparently unobtainable when assuming discrete modes.

Similarly, one can think of more general mode-cost assumptions than ours. In our model, mode cost is linear in commuting time, with fixed and variable components varying only with average speed. Most of the comparative static results obtained from the cost-minimization and utility-maximization models would not be affected by use of a more general mode-cost function, but we would lose some important results. Namely, the unambiguous effects of commuting distance on mode choice and marginal commuting cost would be lost, and these results are used to sign some of the comparative static results of the utility-maximization model, specifically, taste, housing price, and nonwage income effects on mode choice, commute time, and marginal commuting cost. For these reasons, we decided to accept the more restrictive assumption on mode costs.

We would like to introduce other time components into the utility function, in particular, commuting time and working time. This would destroy our model's identity of the wage rate with the cost of time. We suspect this more general utility function would reduce the number of unambiguous results, but it is something we think should be investigated nevertheless.

Additionally, we have ignored many features of modes that commuters value, in particular, comfort features, such as air conditioning, a soft ride, and sound insulation. In doing so, however, we are following standard practice in providing the simplest model that explicates the phenomena in which we are interested, in this case, the effects of wage and nonwage income, tastes for housing, housing prices, and exogenous commuting-cost changes on the household's choice of mode, commuting duration, housing consumption, and residential location.

Other qualitative aspects of various modes, such as reliability and frequency, are often endogenous on commuters' choices through congestion effects. Thus mode choice may be governed by endogenous factors that are absent from our model. Although these variables may be treated as exogenous in our model, and we can therefore show their effects on the model's endogenous variables, to be treated as endogenous variables, however, interactions between modes and such variables must be dealt with in a general equilibrium model, such as that of Mills (1972, pp. 96–108) or, more recently, Anas (1999) and Ross and Yinger (2000).

Despite its restrictive assumptions, however, our extension of the partial equilibrium urban spatial model generates many interesting results and supports the robustness of earlier models in this genre.

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APPENDIX

1. $H_{42} < 0$

The sign of H_{42} is obtained as follows. By rearranging the order in which the Lagrangian function, Equation (6), is differentiated—from x, q, L, k, λ to x, L, q, k, λ—the resulting second-order conditions require

$$\begin{vmatrix} u_{xx} & u_{xL} & -1 \\ u_{Lx} & u_{LL} & -w \\ -1 & -w & 0 \end{vmatrix} > 0$$

But

$$H_{42} = \lambda p_k \begin{vmatrix} u_{xx} & u_{xL} & -1 \\ u_{Lx} & u_{LL} & -w \\ -1 & -w & 0 \end{vmatrix} < 0$$

since $p_k < 0$ and $\lambda = u_x > 0$.

2. $p_{kk}q + C_{kk} > 0$

Expand H along the fourth row, getting $H = -\lambda p_k H_{42} - \lambda (p_{kk}q + C_{kk})H_{44} > 0$. Hence, $p_{kk}q + C_{kk} > -p_k H_{42}/H_{44}$ since $\lambda = u_x > 0$ and

$H_{44} < 0$ by the second-order conditions. But $-p_k H_{42}/H_{44} > 0$ since $p_k < 0$ and $H_{42} < 0$, as shown above.

3. $p_{kk} > 0$

Since $C_{kk} < 0$, from Equation (5), and $q > 0$, then $p_{kk}q + C_{kk} > 0$ implies $p_{kk} > 0$.

4. $H_{54} > 0$

$$H_{52} = \lambda(p_{kk}q + C_{kk}) \begin{vmatrix} u_{xx} & u_{xL} & -1 \\ u_{qx} & u_{qL} & -p \\ u_{Lx} & u_{LL} & -w \end{vmatrix} > 0$$

from the assumption that housing is a normal good. Hence, $\begin{vmatrix} u_{xx} & u_{xL} & -1 \\ u_{qx} & u_{qL} & -p \\ u_{Lx} & u_{LL} & -w \end{vmatrix} > 0$ because $\lambda = u_x > 0$ and $p_{kk}q + C_{kk} > 0$ from above.

$$H_{54} = -\lambda p_k \begin{vmatrix} u_{xx} & u_{xL} & -1 \\ u_{qx} & u_{qL} & -p \\ u_{Lx} & u_{LL} & -w \end{vmatrix} > 0$$

which implies $H_{54} > 0$ since $\lambda = u_x > 0$ and $p_k < 0$.

5. H_{52} *under additive utility*

In the expression for H_{52} above, substitute $u_{xL} = u_{qx} = u_{qL} = u_{Lx} = 0$. Hence

$$H_{52} = \lambda(p_{kk}q + C_{kk}) \begin{vmatrix} u_{xx} & 0 & -1 \\ 0 & 0 & -p \\ 0 & u_{LL} & -w \end{vmatrix} = \lambda(p_{kk}q + C_{kk}) p u_{LL} u_{xx}$$